MorphoGraph and Imagery - Exercises # 3:

originally proposed by Gilles Bertrand

In the wood

The Water and Forest French company has decided to cut down height groves in a forest. The pairwise distances between the groves (in kilometers) are given by the following matrix:

-	1	2	3	4	5	6	7	8
1	-	2.6	4.2	1.8	1.4	3.6	4.0	3.0
2	2.6	-	1.8	3.6	2.4	5.2	4.6	2.2
3	4.2	1.8	-	5.2	3.4	5.0	3.8	2.0
4	1.8	3.6	5.2	-	1.4	3.2	3.0	1.8
5	1.4	2.4	3.4	1.4	-	1.8	2.1	1.6
6	3.6	5.2	5.0	3.2	1.8	-	1.2	2.0
7	4.0	4.6	3.8	3.0	2.1	1.2	-	1.0
8	3.0	2.2	2.0	1.8	1.6	2.0	1.0	-

To complete this enterprise, it is necessary to build a network of roads allowing to go from each grove to any other. The cost for constructing a road is proportional to its length.

Given these data, which network would you propose if your goal is to minimize the construction costs? Justify your solution.

NB: This exercise is extracted and translated from *Optimisation combinatoire, Graphes et programmation linéaire*, Michel Sakarovitch, Herman Ed., 1984.

Spy games

A spy wants to share a secret message with $N-1$ spies. We consider the undirected network whose vertices correspond to these N spies and for which there is an edge between two vertices i and j if and only if a message can be directly exchanged between i and j.

For each edge $\{i, j\}$, we know the probability $p_{i,j}$ that a message sent from i to j is intercepted. We have $p_{i,j} = p_{j,i}$. We assume that message interceptions on distinct edges are independent events in the usual probability sense. How should the message be passed in order to minimize the global interception probability? Give a clear justification to your answer.

Help:

1. Try to reduce the problem to another one that can be solved by a known algorithm
2. The only required knowledge in probability are the following ones:
   - if the probability of the event (E) is p, then the one of $(notP)$ is $1-p$:
   - if (E_1) and (E_2) are two independent events whose respective probabilities are p_1 and p_2, then the probability of the event $(E_1 and E_2)$ is the product $p_1 \times p_2$.

Analysis of particles trajectories

This problem is settled in the framework of automatic analysis of particles trajectories, where the trajectories are acquired by a photographic process in a bubble chamber. Figure (a) represents point cloud resulting from the digitization of a picture. The interpretation of Figure (a) is illustrated in Figure (b): the trajectories have been extracted. Hence, in order to extract trajectories one has to transform (a) into a representation illustrated in figure (c).

1) Propose a series of processings allowing figures like (a) to be transformed into graphs that model figures like (c). The original data, corresponding to (a), are given as a set of n points $E = \{P_1, \ldots, P_n\}$, with $P_i = (x_i, y_i)$, x_i and y_i being the coordinates of point P_i. You are strongly encouraged to use methods seen in class! You can present your process in plain English (no need for algorithmic schemes). Illustrate (the successive steps of) your process on the figures given below.

2) What is the time complexity of your method?

3) What kind of noise cannot be avoided by your method? Propose a way to improve your method.

(a) (b) (c)