Mathematical morphology for hyperspectral images

Filtering and segmentation in a supervised framework

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Séminaire "Morphologie mathématique, structures et connexions"

70ème anniversaire de Jean Serra

ESIEE (Marne-La-Vallée) - 2 avril 2010

Acknowledgements

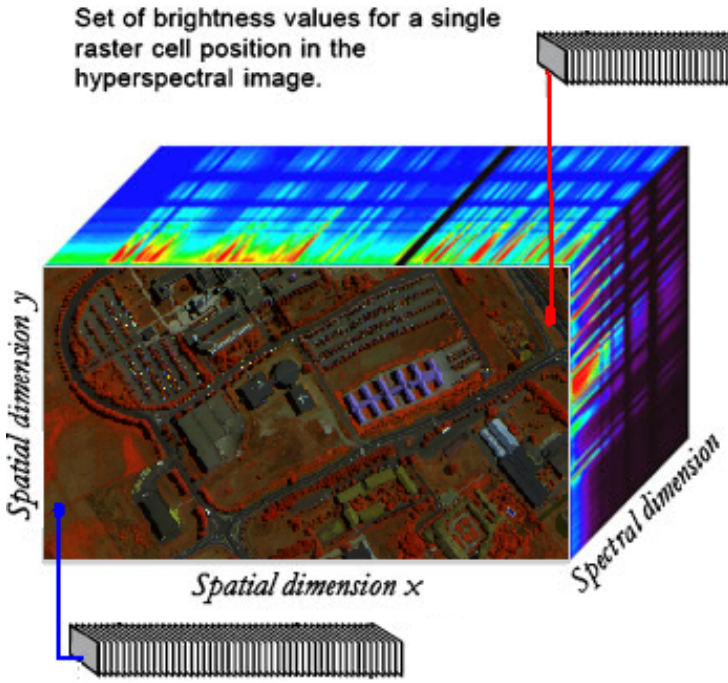
Thanks to my Ph.D. student

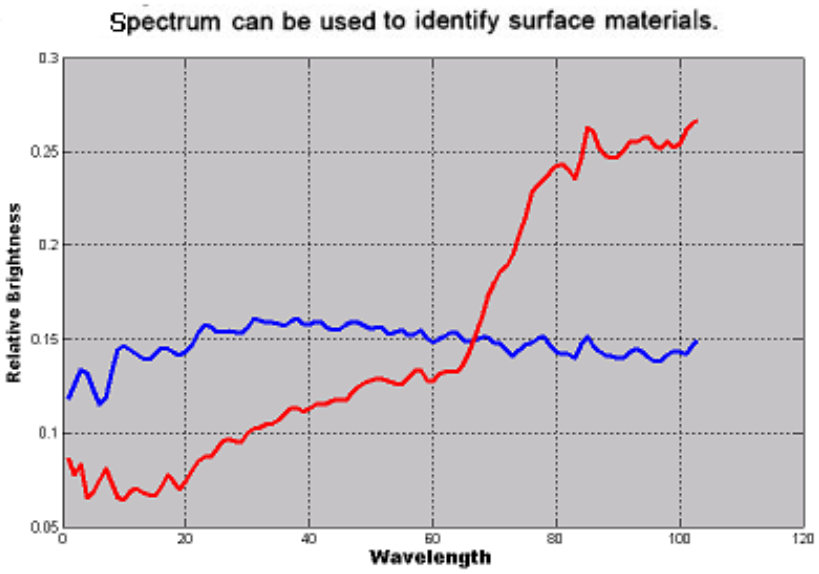
Santiago Velasco-Forero

an excellent collaborator in the development of these works...

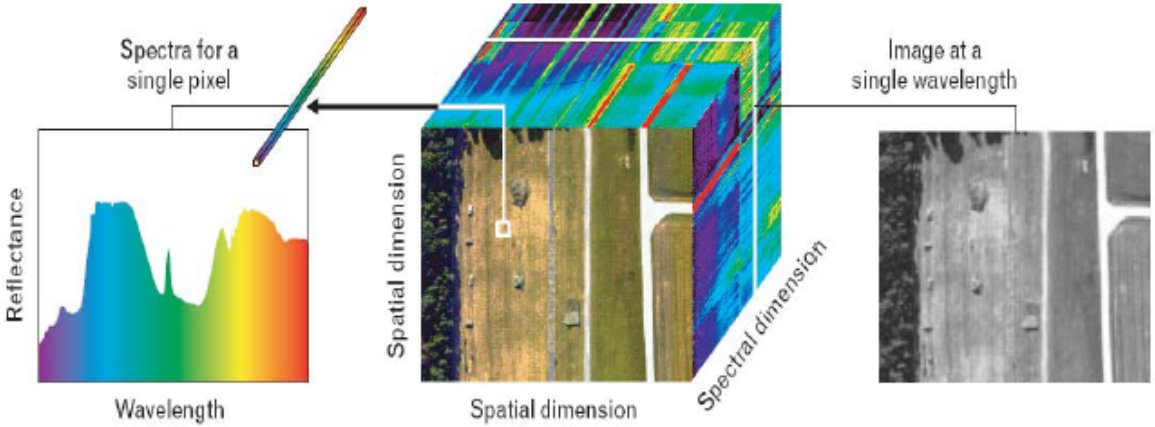
Plan

Hyperspectral images: A spectral signature for each pixel of the image





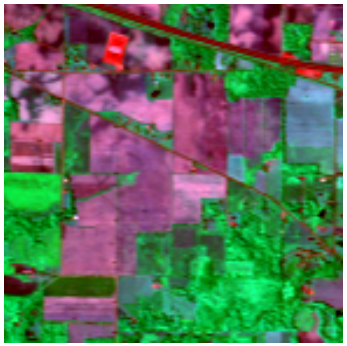
Hyperspectral images: Vectorial images



Notation: Let $\mathbf{f}_\lambda(\mathbf{x}) = \{f_{\lambda_j}(\mathbf{x})\}_{j=1}^{L}$ be a hyperspectral image

- $\mathbf{f}_\lambda : E \longrightarrow \mathcal{T}^L$;
- $\mathbf{x} = (x, y) \in E \subset \mathbb{Z}^2$ are the spatial coordinates of a vector pixel;
- Each pixel has associated a vector which corresponds to a spectrum: $\mathbf{f}_\lambda(\mathbf{x}_i) = \mathbf{s}_k$;
- Space of spectral values is $\mathcal{T}^L \subset \mathbb{R}^L$;
- Scalar image $f_{\lambda_j}(\mathbf{x})$ corresponds to the channel or band j, $j \in \{1, 2, \ldots, L\}$.

Hyperspectral images: Examples



(1) "Indian Pines"

(2) "Pavia"

(1) Hyperspectral image of the Indian Pines (200 spectral bands in the 400-2500 nm range, 145x145 pixels), obtained by the AVIRIS sensor.
(2) Airbone image from the ROSIS-3 optical sensor of the University of Pavia (103 spectral bands of 340x610 pixels).

Supervised paradigm for hyperspectral image analysis

- Fundamental assumption: $\mathbf{f}_\lambda(\mathbf{x})$ is composed of K spectral classes:

$$\{C_1, C_2, \cdots, C_K\}.$$

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$$C_k \equiv \{\mathbf{s}_{1,k}, \mathbf{s}_{2,k}, \cdots, \mathbf{s}_{n_k,k}\},$$

such that $\mathbf{s}_{i,k} \in \mathcal{T}^L$.

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- **Our methodological aim: Extension of mathematical morphology operators to hyperspectral images consistent with the supervised paradigm**

Basic notions

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- A space $\mathcal{L}$ endowed with a partial order $\leq$ is called a *complete lattice*, if for every subset $\mathcal{H} \subseteq \mathcal{L}$ have both supremum (join) $\bigvee \mathcal{H}$ and infimum (meet) $\bigwedge \mathcal{H}$.

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- A *minimum (smallest)* $n \in \mathcal{H}$ is an element contained in all other elements of $\mathcal{H}$, i.e., $l \in \mathcal{H} \Rightarrow n \leq l$. We denote the minimum of $\mathcal{L}$ by $\perp$.
- A *maximum (largest)* n in $\mathcal{H}$ is an element that contains every element of $\mathcal{H}$, i.e., $l \in \mathcal{H} \Rightarrow l \leq n$. We denote the maximum of $\mathcal{L}$ by $\top$.

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- Note that $\leq_h$ preserves reflexivity ($r \leq_h r$) and transitivity ($r_1 \leq_h r_2$ and $r_2 \leq_h r_3 \Rightarrow r_1 \leq_h r_3$)
- But is not a total ordering: An equivalence class is defined by $\mathcal{L}[z] = \{r \in R | h(r) = z\}$.

Families of h-mappings

$h : \mathbb{R}^p \to \mathcal{L} \equiv \mathbb{R} \cup \{-\infty, +\infty\}$

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- Based on (local) projections: Under the "cluster assumption" (local structures in $\mathbb{R}^p$), different projections can be obtained per cluster, i.e.,

$$h(\mathbf{s}) = \sum_{i=1}^{p} \lambda_{\mathbf{s}}^{i} s^{i}$$

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- Based on (local) adaptive distances: Given the reference (or training) subset $T \subset \mathbb{R}^p$, $T = \{\mathbf{t}_1, \ldots, \mathbf{t}_{|T|}\}$, with $\mathbf{t}_i \in \mathbb{R}^p, \forall i$, we have

$$h(\mathbf{s}) = \sum_{i=1}^{|T|} \lambda_{\mathbf{s}}^{i} \phi(\mathbf{t}_i, \mathbf{s})$$

where $\phi : \mathbb{R}^p \times \mathbb{R}^p \rightarrow \mathbb{R}^+$ is a kernel-induced distance and weights $\lambda_{\mathbf{s}}^{i}$ are fitted for each vector $\mathbf{s}$ in $\mathbb{R}^p$

h-supervised ordering based on kernelized distances

- Contribution: To introduce a supervised ordering formulation, based on both background B and foreground F training sets, which allows an adequate interpretation of dual morphological operators.

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- Basic assumption: Given a set R and the subsets $B, F \subset R$, such that $B \cap F = \emptyset$, we define a h-ordering that satisfies the conditions:

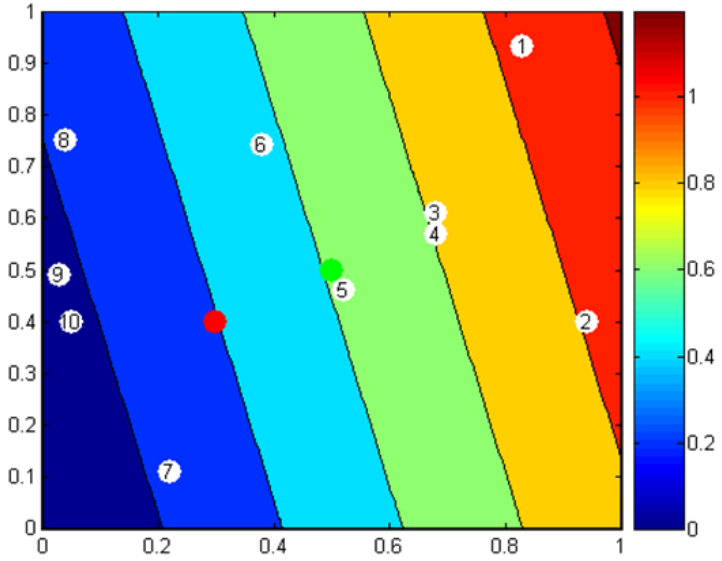
$$h(\mathbf{s}) = \perp \text{ if } \mathbf{s} \in B$$

and

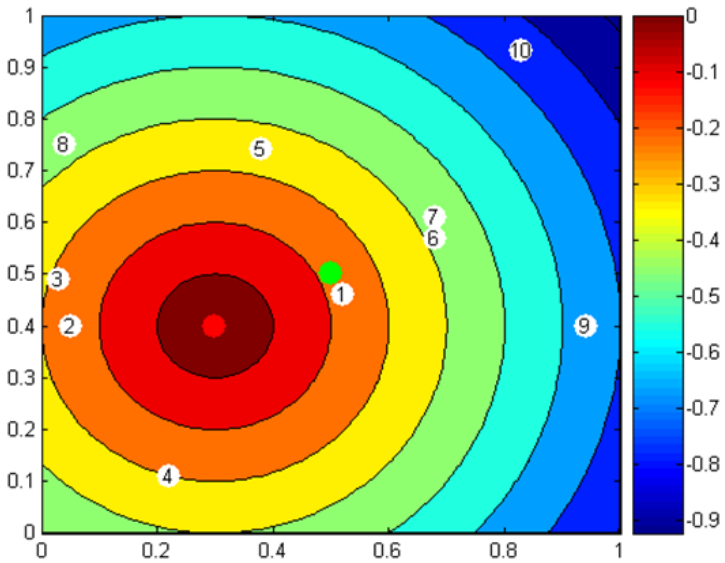
$$h(\mathbf{s}) = \top \text{ if } \mathbf{s} \in F$$

where $\perp, \top$ are the smallest and largest element in the lattice $\mathcal{L}$.

Examples of h-orderings in $\mathbb{R}^2$



Unsupervised ordering using first principal component: $h(\mathbf{s}) = \sum_{i=1}^{2} \lambda^i s_i$



Supervised ordering using a single reference $\mathbf{f}$ (red circle): $h_{\mathbf{f}}(\mathbf{s}) = \mathbf{K}(\mathbf{s}, \mathbf{f})$

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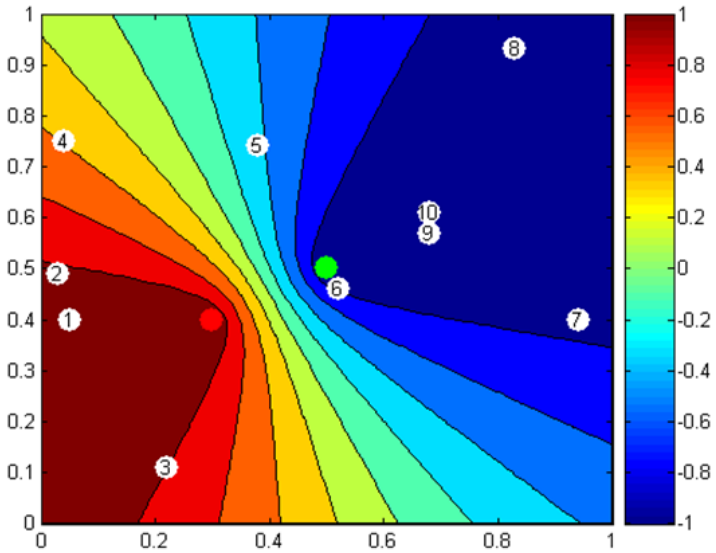
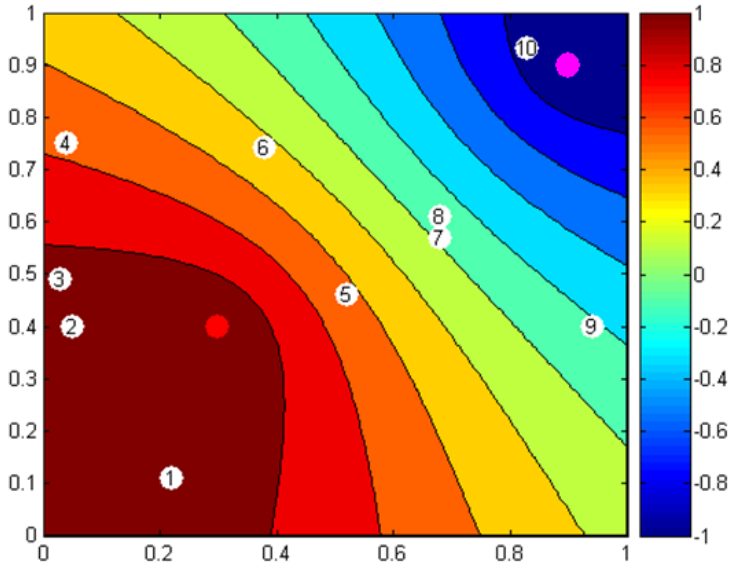


Supervised ordering using based on

$F = \mathbf{f}$ (red circle) and $B = \mathbf{b_1}$(green circle):

$$h_{[\mathbf{f},\mathbf{b_1}]}(\mathbf{s}) = \frac{K(\mathbf{f},\mathbf{s}) - K(\mathbf{b_1},\mathbf{s})}{K(\mathbf{s},\mathbf{s}) - K(\mathbf{f},\mathbf{b_1})}$$



Supervised ordering using based on

$F = \mathbf{f}$ (red circle) and $B = \mathbf{b_2}$(purple circle):

$$h_{[\mathbf{f},\mathbf{b_2}]}(\mathbf{s}) = \frac{K(\mathbf{f},\mathbf{s}) - K(\mathbf{b_2},\mathbf{s})}{K(\mathbf{s},\mathbf{s}) - K(\mathbf{f},\mathbf{b_2})}$$

Hyperspectral erosion and dilation

- Total ordering: The function $h_{[F,B]}(\mathbf{s})$ yields a partial ordering in $\mathbb{R}^p$, but in practical applications for hyperspectral image processing, a total ordering is required: the total ordering is induced including a lexicographic order in $\mathcal{L}[z]$, for all z.

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- Erosion and dilation: For the hyperspectral image $\mathbf{f}_\lambda(\mathbf{x})$, given the ordering mapping $h_{[F,B]}(\mathbf{s})$ and the structuring element S

$$\begin{cases} \varepsilon_{\langle h_{[\boldsymbol{F},\boldsymbol{B}]},S\rangle}(\mathbf{f}_\lambda)(\mathbf{x}) = \{\mathbf{f}_\lambda(\mathbf{y}) : h_{[F,B]}\left(\mathbf{f}_\lambda(\mathbf{y})\right) = \bigwedge \left[h_{[F,B]}\left(\mathbf{f}_\lambda(\mathbf{z})\right)\right], \mathbf{z} \in S(\mathbf{x}))\}, \\ \delta_{\langle h_{[\boldsymbol{F},\boldsymbol{B}]},S\rangle}(\mathbf{f}_\lambda)(\mathbf{x}) = \{\mathbf{f}_\lambda(\mathbf{y}) : h_{[F,B]}\left(\mathbf{f}_\lambda(\mathbf{y})\right) = \bigvee \left[h_{[F,B]}\left(\mathbf{f}_\lambda(\mathbf{z})\right)\right], \mathbf{z} \in S(\mathbf{x}))\}. \end{cases}$$

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- Duality:

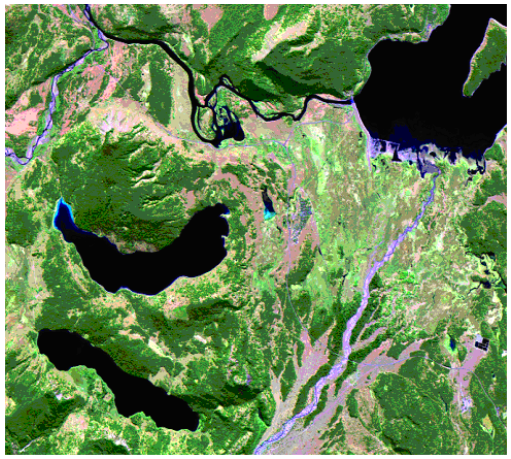
$$\varepsilon_{\langle h_{[\boldsymbol{F},\boldsymbol{B}]},S\rangle}(\mathbf{f}_\lambda) = \delta_{\langle h_{[\boldsymbol{B},\boldsymbol{F}]},S\rangle}(\mathbf{f}_\lambda)$$

and

$$\varepsilon_{\langle h_{[\boldsymbol{F},\boldsymbol{B}]},S\rangle}(\mathbf{f}_\lambda) = \delta_{\langle -h_{[\boldsymbol{F},\boldsymbol{B}]},S\rangle}(\mathbf{f}_\lambda)$$

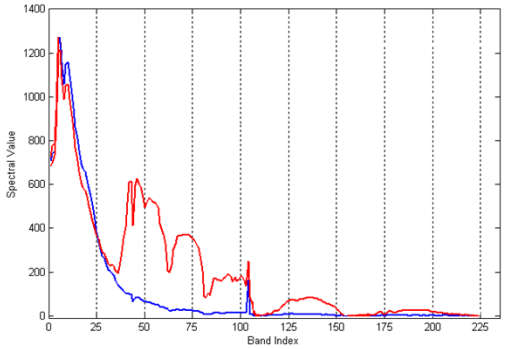
Hyperspectral morphological processing

Hyperspectral erosion and dilation

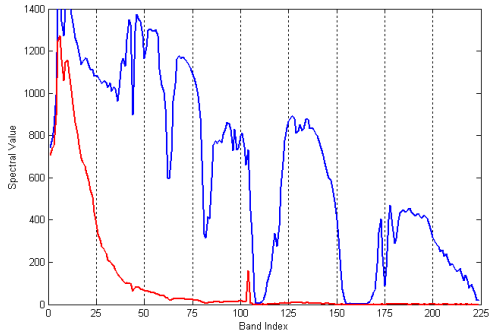


Original image

$\mathbf{f}_\lambda(\mathbf{x})$



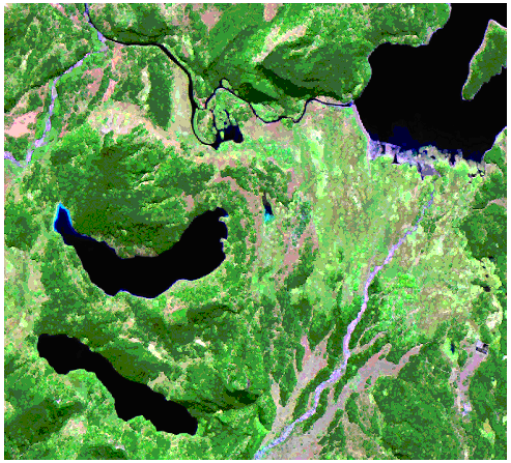
$F_1 = \{\mathbf{s}_{water}\}$ (in blue), $B_1 = \{\mathbf{s}_{vegetation}\}$ (in red)



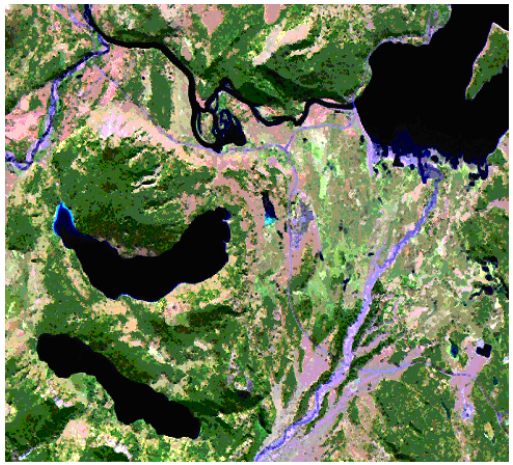
$F_2 = \{\mathbf{s}_{land}\}$ (in blue), $B_2 = \{\mathbf{s}_{water}\}$ (in red)

Hyperspectral morphological processing

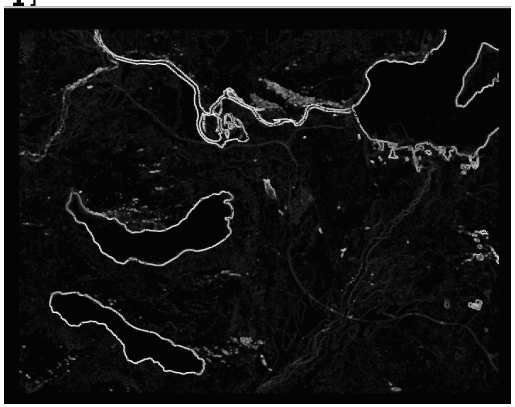
Hyperspectral erosion and dilation



Unitary hexagonal erosion $\varepsilon_{\langle h_{[F_1,B_1]},H\rangle}(\mathbf{f}_\lambda)(\mathbf{x})$



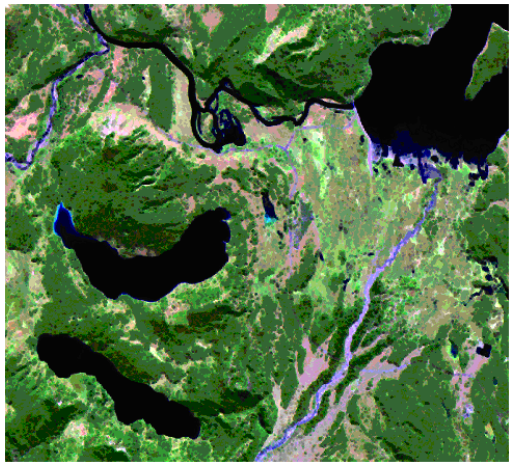
Unitary hexagonal dilation $\delta_{\langle h_{[F_1,B_1]},H\rangle}(\mathbf{f}_\lambda)(\mathbf{x})$



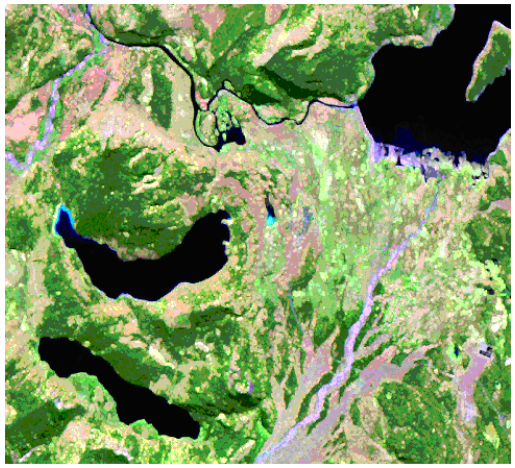
Morphological gradient $\delta_{\langle h_{[F_1,B_1]},H\rangle}(\mathbf{f}_\lambda)(\mathbf{x}) -_h \varepsilon_{\langle h_{[F_1,B_1]},H\rangle}(\mathbf{f}_\lambda)(\mathbf{x})$

Hyperspectral morphological processing

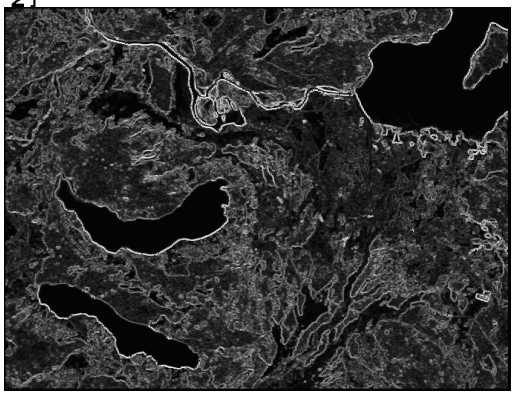
Hyperspectral erosion and dilation



Unitary hexagonal erosion $\varepsilon_{\langle h_{[F_2,B_2]},H\rangle}(\mathbf{f}_\lambda)(\mathbf{x})$



Unitary hexagonal dilation $\delta_{\langle h_{[F_2,B_2]},H\rangle}(\mathbf{f}_\lambda)(\mathbf{x})$



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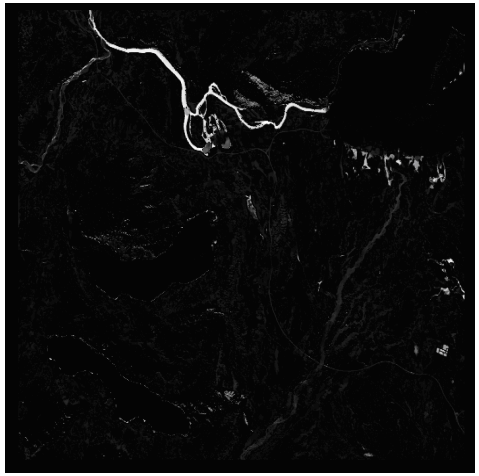
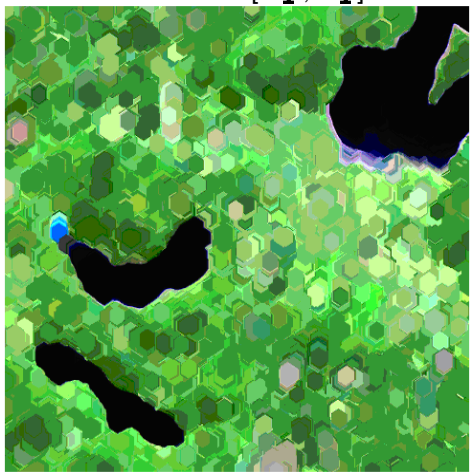
Hyperspectral morphological processing

Hyperspectral opening

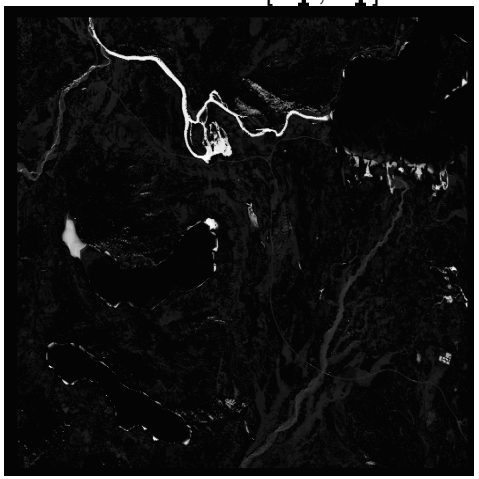
Opening Hex. 4 $\gamma_{\langle h_{[F_1,B_1]},10H\rangle}(\mathbf{f}_\lambda)(\mathbf{x})$



Top-Hat $\mathbf{f}_\lambda)(\mathbf{x}) -_h \gamma_{\langle h_{[F_1,B_1]},10H\rangle}(\mathbf{f}_\lambda)(\mathbf{x})$



Opening Hex. 10 $\gamma_{\langle h_{[F_1,B_1]},10H\rangle}(\mathbf{f}_\lambda)(\mathbf{x})$

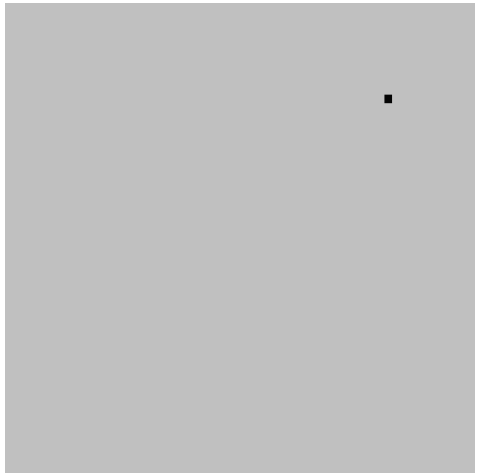


Top-Hat $\mathbf{f}_\lambda)(\mathbf{x}) -_h \gamma_{\langle h_{[F_1,B_1]},10H\rangle}(\mathbf{f}_\lambda)(\mathbf{x})$

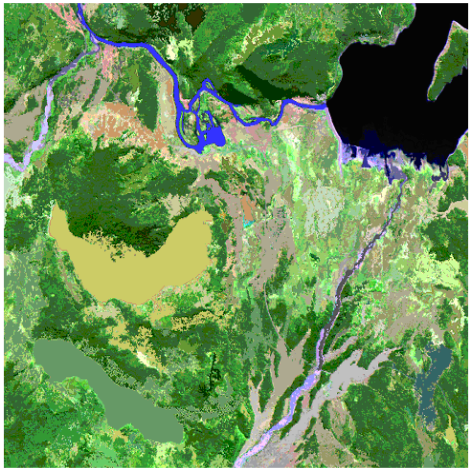
Hyperspectral morphological processing

Hyperspectral swamping (geodesic reconstruction)

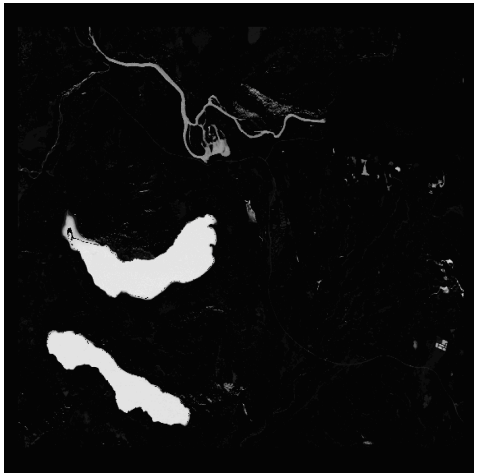
Original $\mathbf{f}_\lambda(\mathbf{x})$



Marker $\mathbf{m}_\lambda(\mathbf{x})$



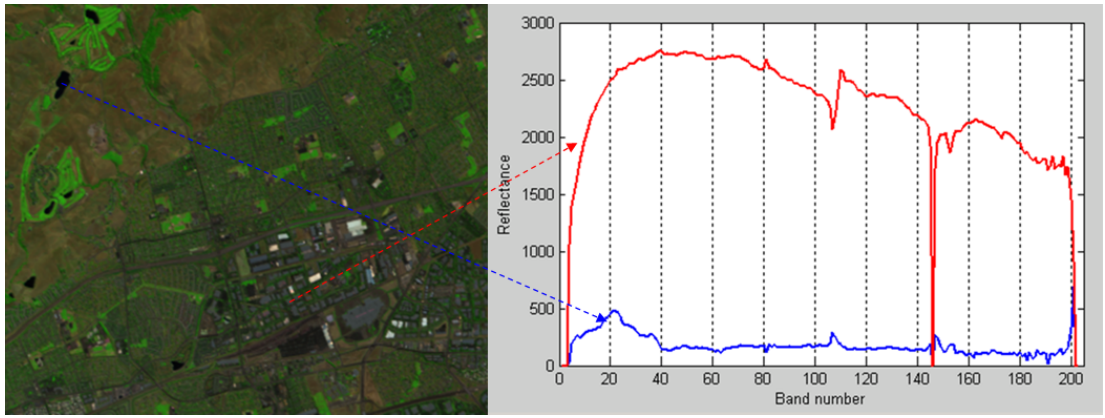
Geodesic. Recons. $\gamma^{rec}_{\langle h_{[F_1,B_1]}\rangle}(\mathbf{f}_\lambda,\mathbf{m}_\lambda)(\mathbf{x})$



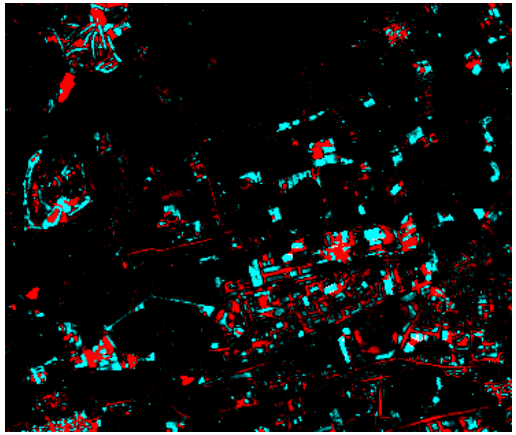
Residue $\mathbf{f}_\lambda)(\mathbf{x}) -_h \gamma^{rec}_{\langle h_{[F_1,B_1]}\rangle}(\mathbf{f}_\lambda,\mathbf{m}_\lambda)(\mathbf{x})$

Hyperspectral morphological processing

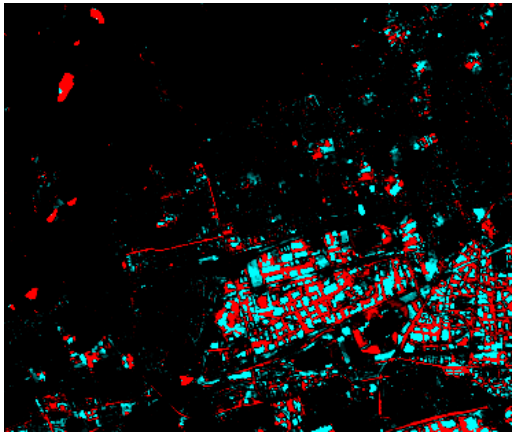
Spectral structure extraction



Original $\mathbf{f}_\lambda(\mathbf{x})$ and $F = \mathbf{f}$, $B = \mathbf{b}$



$\rho^{+}_{h_{[F,\emptyset],5H}}(\mathbf{f}_\lambda)$, $\rho^{-}_{h_{[F,\emptyset],5H}}(\mathbf{f}_\lambda)$



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Hyperspectral morphological processing

Spatial-driven supervised classification

Classif.

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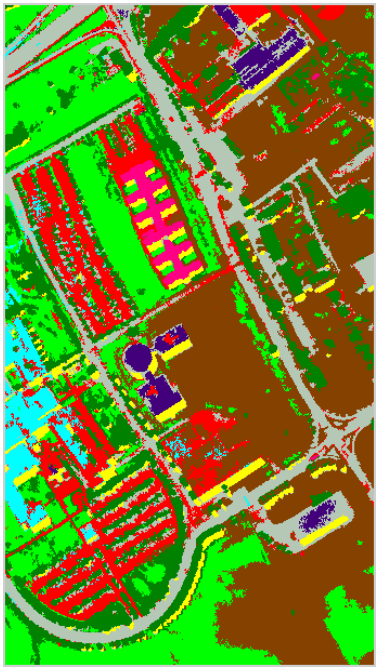

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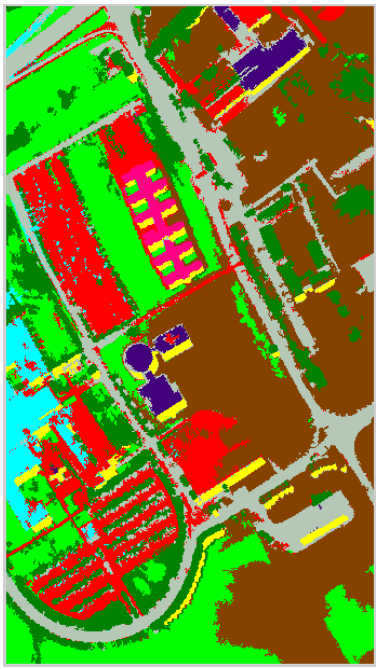

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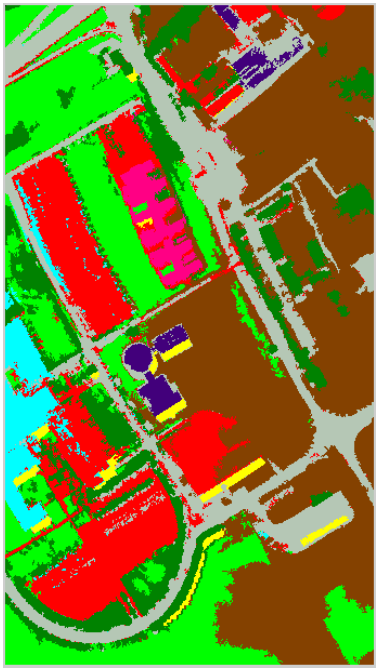

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Semi-supervised segmentation using regionalized stochastic watershed

- Aim: Semi-supervised segmentation of hyperspectral images
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- Context: In high spatial resolution images, there are many complex structures at various scales and the definition of a single segmentation is a challenging difficult problem
- Contribution: MonteCarlo simulations of regionalized germs, according to a membership probability map, for the computation of the probability of contours using the stochastic watershed

Modelling Membership Probability Map of Spectral Classes

- Standard assumption on hyperspectral imaging: each spectral class follow a normal distribution $C_k \equiv \mathcal{N}(\mu_k, \Sigma_k)$. Hence, using Bayesian terminology, the Gaussian conditional density per class is given by

$$\Pr(\mathbf{s}_i \mid C_k) = \frac{1}{(2\pi)^{\frac{L}{2}} |\Sigma_k|^{\frac{L}{2}}} e^{-\frac{1}{2}(\mathbf{s}_i - \mu_k)^T \Sigma_k^{-1} (\mathbf{s}_i - \mu_k)}$$

- According to the Bayes Decision Rule, the maximum a posteriori discriminant function becomes (eliminating constant terms, taking natural logs and assuming also equiprobable priors)

$$g_k(\mathbf{s}_i) = \Pr(C_k \mid \mathbf{s}_i) \approx -\frac{1}{2}\left[(\mathbf{s}_i - \mu_k)^T \Sigma^{-1} (\mathbf{s}_i - \mu_k)\right]$$

Modelling Membership Probability Map of Spectral Classes

- Membership Probability Map (MPM) of the class C_k, $\pi_k^M(\mathbf{x}) : E \rightarrow \mathbb{R}_+$, is defined by

$$\pi_k^M(\mathbf{x}) = \frac{\exp\left(\frac{-\frac{1}{2}\left[(\mathbf{f}_\lambda(\mathbf{x})-\mu_k)^T \Sigma_k^{-1}(\mathbf{f}_\lambda(\mathbf{x})-\mu_k)\right]}{\sigma_{MPM}}\right)}{\sum_{\mathbf{y}\in E}\exp\left(\frac{-\frac{1}{2}\left[(\mathbf{f}_\lambda(\mathbf{y})-\mu_k)^T \Sigma_k^{-1}(\mathbf{f}_\lambda(\mathbf{y})-\mu_k)\right]}{\sigma_{MPM}}\right)}.$$

- A high value of $\pi_k(\mathbf{x})$ implies that the image value spectrum $\mathbf{f}_\lambda(\mathbf{x})$ has a high probability to belongs to the class C_k.
- The parameter σ_{MPM} allows to introduce a scaling regularization of the distance values ($\sigma_{MPM} = 0.1$ produces a good trade-off).
- In the experiments of this paper we use only 10 spectral samples as training set $\Rightarrow \Sigma_k$ is the identity matrix (the computation of the inverse of covariance matrix Σ_k^{-1} makes no sense and it may introduce important errors in the distance)

Regionalized Random Germs Simulations from MPM

- Regionalized Poisson Points: Given a density function variable in the space $\theta(\mathbf{x})$ (being a measurable function in $\mathbb{R}^d$, with positive values), the number of points falling in a borel set B according to a θ follows a Poisson distribution of parameter $\theta(D)$, i.e.,

$$\Pr\{N(D) = n\} = e^{-\theta(D)} \frac{(-\theta(D))^n}{n!}.$$

  with $\theta(D) = \int \theta(\mathbf{x}) d\mathbf{x}$.
- In such a case, if $N(D) = n$, the n are independently distributed over D with the probability density function $\widehat{\theta}(\mathbf{x}) = \theta(\mathbf{x})/\theta(D)$.
- Aim: Generating a non-uniform distribution of germs, with a regionalized pdf associated to class k such that $\widehat{\theta}(\mathbf{x}) \equiv \pi_k(\mathbf{x})$.

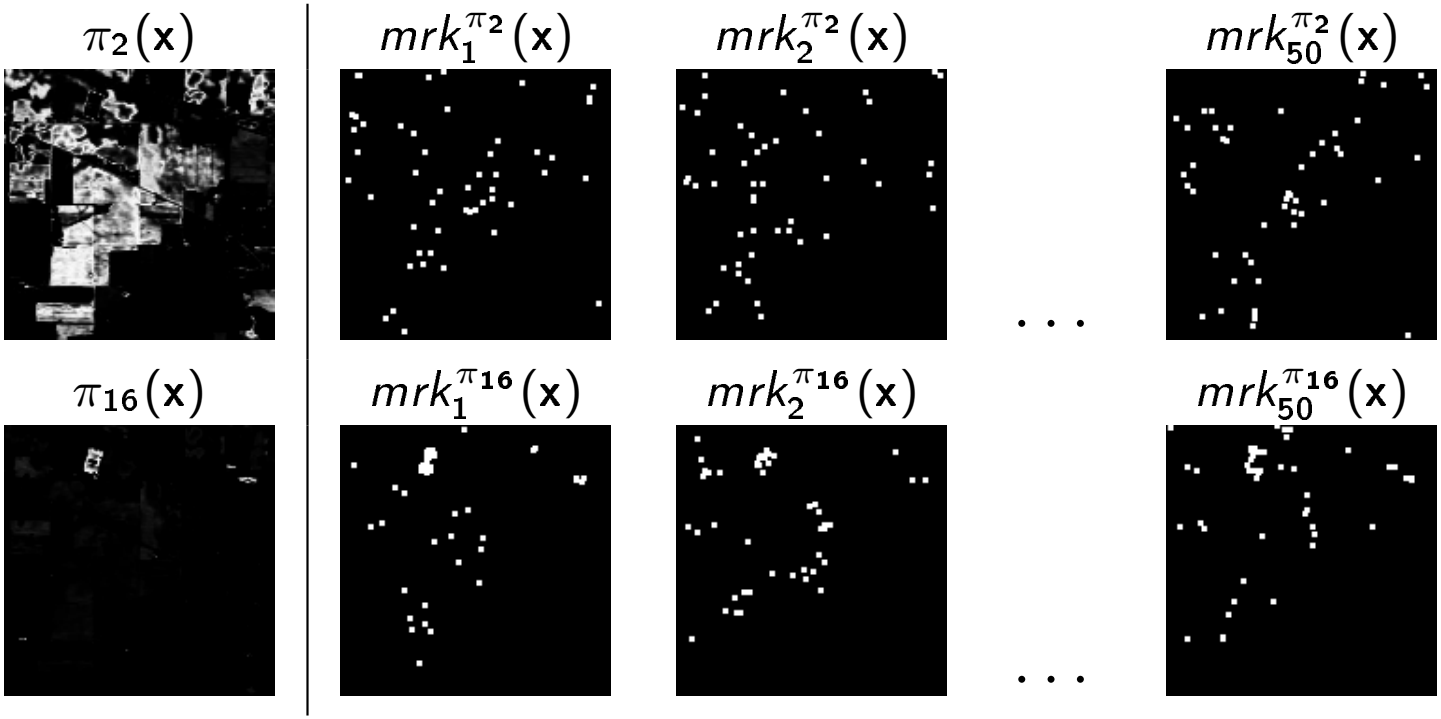
Regionalized Random Germs Simulations from MPM

- Algorithm to simulate in $m(\mathbf{x}_i)$ a realization of N independent random germs distributed according to $\pi_k(\mathbf{x})$ using an inverse transform sampling method:

1. Initialization: $m(\mathbf{x}_i) = 0 \ \forall \mathbf{x}_i \in E$; $P = \text{Card}(E)$
2. Compute cumulative distribution function: $cdf(\mathbf{x}_i) = \frac{\sum_{k \leq i} \pi_k(\mathbf{x}_k)}{\sum_{k=1}^{P} \pi_k(\mathbf{x}_k)}$
3. for $j = 1$ to N
4. $\quad r_j \sim \mathcal{U}(1, P)$
5. $\quad$ Find the value s_j such that $r_j \leq cdf(\mathbf{x}_{s_j})$.
6. $\quad m(\mathbf{x}_{s_j}) = 1$

Regionalized Random Germs Simulations from MPM

- Examples:



Computation of pdf of contours

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- Estimation of the probability density of contours of spectral band λ_j with respect to the class C_k, i.e.,

$$pdf_{\lambda_j}^{C_k}(\mathbf{x}) = \frac{1}{M}\sum_{i=1}^{M} WS(\varrho(f_{\lambda_j}), mrk_i^{\pi_k})(\mathbf{x}) * K(\mathbf{x}; \sigma_{spatial}).$$

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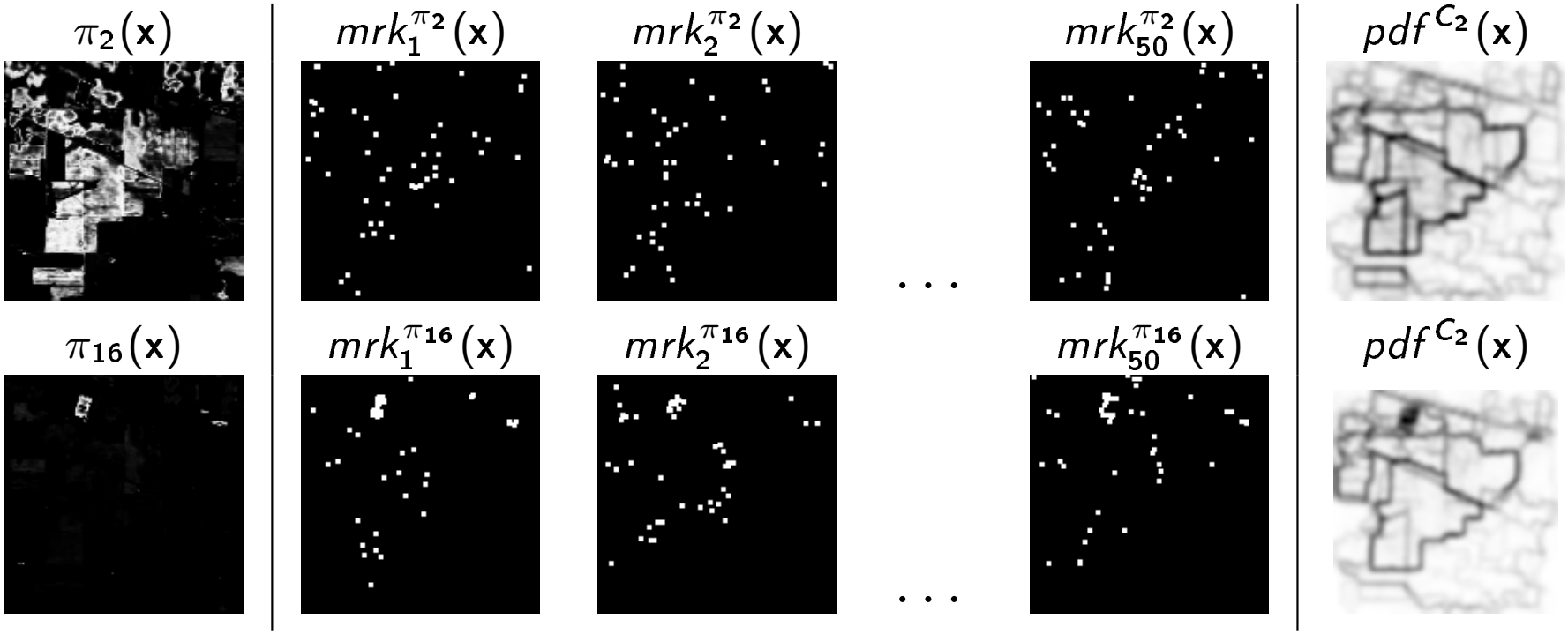
- Probability density of contours of hyperspectral image w.r.t. the class C_k is obtained as the kernelized sum of the marginal pdf's of the spectral bands, i.e.,

$$\{\mathbf{f}_\lambda(\mathbf{x}), C_k\} \mapsto pdf^{C_k}(\mathbf{x}) = \frac{1}{L}\sum_{j=1}^{L} pdf_{\lambda_j}^{C_k}(\mathbf{x}) * K(\lambda; \sigma_{spectral}).$$

Computation of pdf of contours

- Examples:



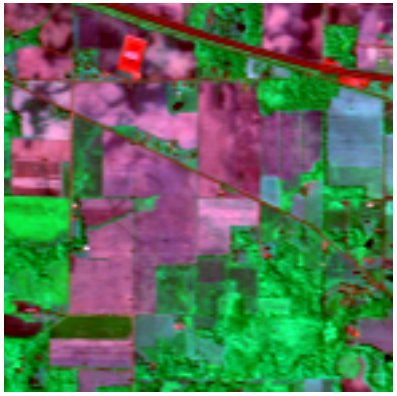
Hierarchical Segmentation

- The $pdf^S(\mathbf{x})$ could be directly thresholded in order to obtain the most prominent contours
- However, the results are only pieces of contours (not enclosing regions).
- There is not an optimal threshold to separate the classes of contours.
- An alternative technique to segment automatically the pdf of the hyperspectral image in significant closed regions is to apply a morphological hierarchical algorithm. Mainly, two hierarchical techniques can be distinguished:
  i) Non-parametric waterfalls algorithm;
  ii) Hierarchies based on extinction values, which allows to select the minima used in the watershed according to morphological criteria (dynamics, surface area and volume).
- By selecting a particular level of the hierarchy, contours of the regions having a higher probability to belong to the final segmentation than the regions appearing in lower levels are obtained.

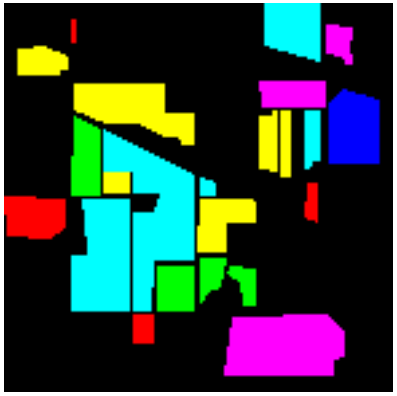
Hierarchical Segmentation

- Examples of hierarchical segmentation

Image

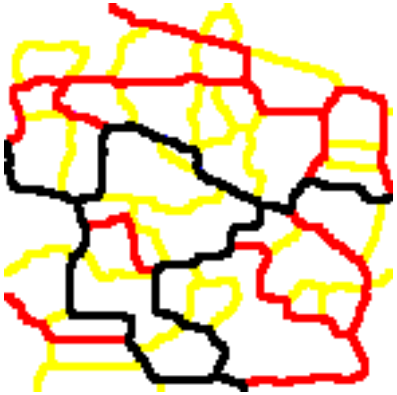


Class 6 - Green

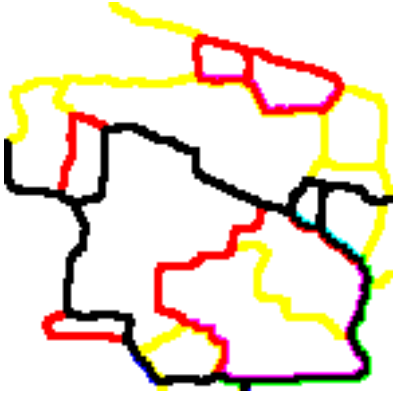


$pdf^{C_6}(\mathbf{x})$

Waterfalls-based
3 levels



Dynamics-based
$5, 10, 20$ regions



Volumic-based
$5, 10, 20$ regions

Hierarchical Segmentation

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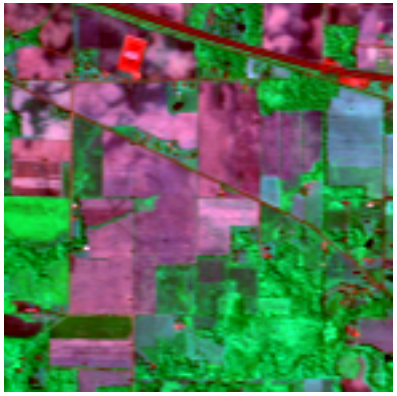
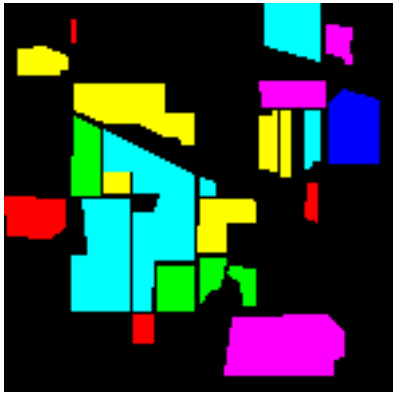
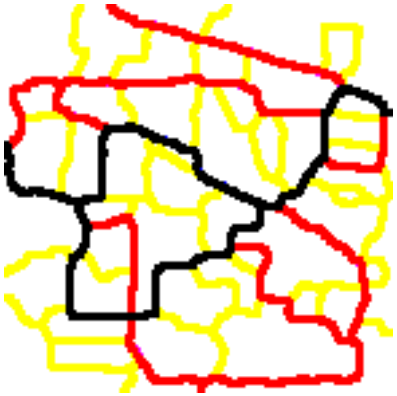


Class 8 - Blue



$pdf^{C_8}(\mathbf{x})$

Waterfalls-based
3 levels



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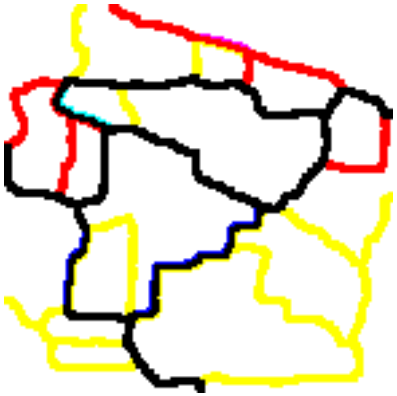
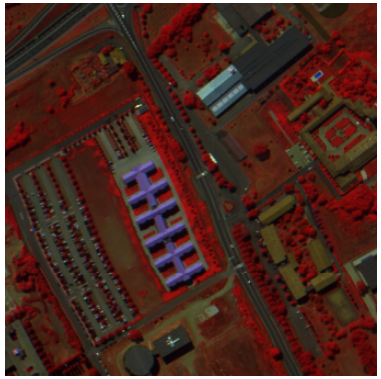


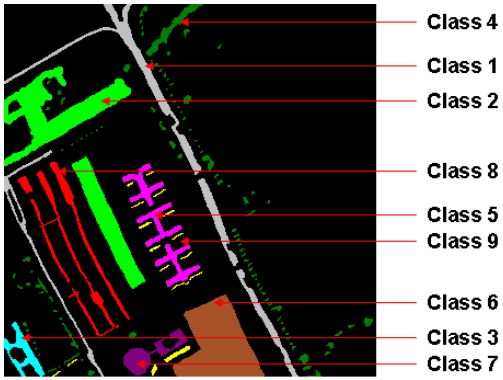
Hierarchical Segmentation

- Examples of dynamics-based hierarchical segmentation

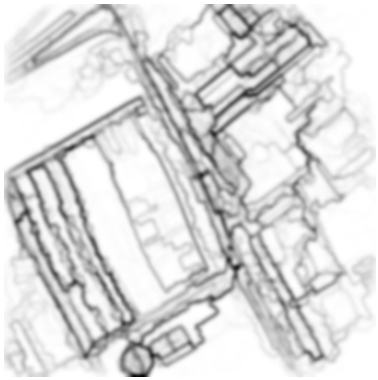
Image



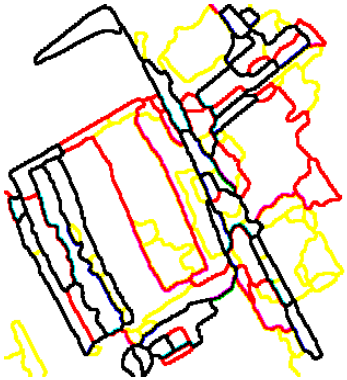
Ground Truth



$pdf^{C_7}(\mathbf{x})$



$25, 50, 100$ regions



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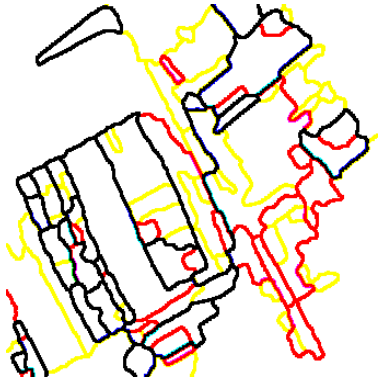
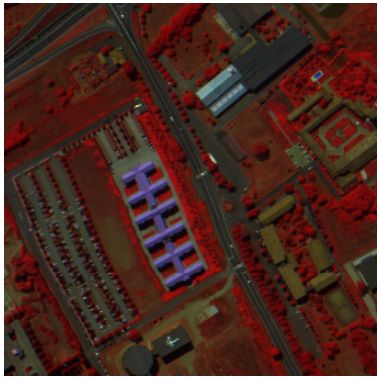


Hierarchical Segmentation

- Examples of dynamics-based hierarchical segmentation

Image



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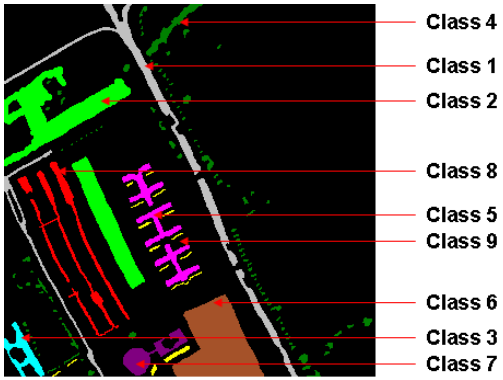
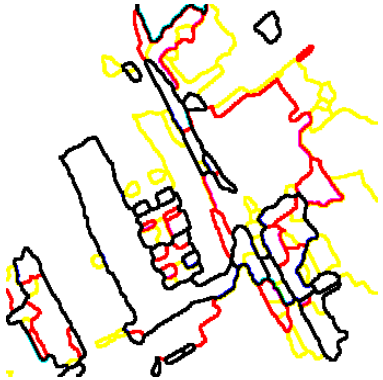


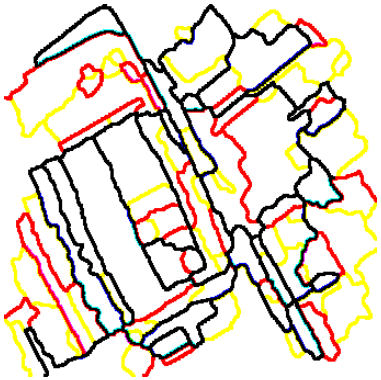
$pdf^{C_9}(\mathbf{x})$

25, 50, 100 regions



Unsuperv. $pdf(\mathbf{x})$

25, 50, 100 regions



Conclusions and Perspectives

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  - J. Serra. **Anamorphoses and Function Lattices (Multivalued Morphology)**. In *Mathematical Morphology in Image Processing*, E. Dougherty, Marcel-Dekker, 483-523, 1992.
  - J. Serra, M. Mlynarczuk. **Morphological merging of multidimensional data.** *Proc. of STERMAT'00*, pp. 485-390, 2000.
  - J. Serra. **The "False Colour" Problem.** *Proc. of the ISMM'09*, LNCS Vol. 5720, 13-23, 2009.

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- **Next:**
  - (Pseudo-)mathematical morphology for non Euclidean spaces: Riemannian manifolds, Statistical manifolds
  - Banach Lattices and Operators

More details...

- S. Velasco-Forero and J. Angulo. "**Supervised ordering in R^p: Application to morphological processing of hyperspectral images**". Submitted to *IEEE Transactions on PAMI*, 2010.
- J. Angulo, and S. Velasco-Forero. "**Semi-supervised hyperspectral image segmentation using regionalized stochastic watershed**". In *Proc. of SPIE symposium on Defense, Security, and Sensing: Algorithms and Technologies for Multispectral, Hyperspectral, and Ultraspectral Imagery XVI*, SPIE Vol. 7695, Orlando, United States, April 2010.