

# Combined optimization of electrical and mechanical parameters of in-plane and out-of-plane gap-closing electrostatic Vibration Energy Harvesters (VEHs)

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## Abstract

This paper presents a simple analytical method to optimize the efficiency of two types of electrostatic Vibration Energy Harvesters (VEH): the out-of-plane (OPGC) and in-plane (IPGC) gap-closing converters. For the first time the electrical and mechanical behaviours of the transducer are addressed simultaneously, while a voltage limitation on the transducer's terminals is set to prevent any damage in the conditioning electronic. The presented work allows to the designer to determine the best strategy depending on whereas the system is passive or able to be self-adapted to the external vibrations parameters. The calculations are validated by VHDL-AMS/ELDO simulations.

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## 1. Introduction

In order to improve the efficiency of an electrostatic Vibration Energy Harvester (VEH), it's common to design a transducer with the best maximal-to-minimal capacitance ratio and to initialize it with the highest voltage as possible. Indeed, the maximal power that can be harvested for a VEH working at constant charge [1]:

$$P'_{h\_max} = \frac{U_0^2}{2} C_{max} \left( \frac{C_{max}}{C_{min}} - 1 \right) f_{elec} \quad (1)$$

where  $C_{max}$  and  $C_{min}$  are the maximal and the minimal values of the transducer's capacitance  $C_{tran}$ ,  $U_0$  is the initial voltage applied on  $C_{tran}$  at  $C_{max}$  and  $f_{elec}$  is the frequency of  $C_{tran}$ 's variations. However the designer has to consider that if a high initial voltage leads to a high electromechanical coupling, it also limits the capacitance variation due to the phenomenon of electrostatic instability (pull-in) [2]. Moreover, a too high capacitance variation could lead to voltages too high for the surrounding electronics. Therefore to maximize the efficiency of an electrostatic VEH, a compromise between the pre-charge voltage, the displacement range of the seismic mass and the maximal

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capacitance value of the transducer is required. In this work, we have performed an accurate analytical modelization of the Out-Of-Plane-Gap-Closing (OPGC) and In-Plane-Gap-Closing (IPGC) architectures [3] in order to determine the best design in term of harvested power for practical implementation, i.e. we have taken into account a voltage limitation in the conditioning electronic and we have considered if the system is self-adapted or not with the vibration's amplitude changes.

## 2. Description of the transducers

The structures of the OPGC and IPGC VEHS are represented in Fig. 1. The transducer's capacitance  $C_{tran}$  is composed of a variable part  $C_{var}$  and a constant part  $C_{par}$  in parallel to  $C_{var}$ .  $d_0$  is the gap between the electrodes when no voltage or acceleration is applied to the system. For our demonstration we will consider a mobile mass made of 400  $\mu\text{m}$ -thick bulk silicon, having an area of 1  $\text{cm}^2$ . In the IPGC architecture, 2 mm x 30  $\mu\text{m}$  combs are etched on both sides of the mobile mass. The mechanical resonance is 200 Hz,  $C_{par}$  is equal to 10 pF and the thickness of stoppers  $t_s$  is 1  $\mu\text{m}$  by default ( $= t_{s\_min}$ ). The maximal voltage allowed in the system is 60 V.

When the pre-charge voltage is lower than the pull-in voltage  $U_{pi}$  the system has two equilibrium positions: one stable and one unstable, situated at  $x_{eq\_stable}$  and  $x_{eq\_unstable}$ , so that  $d_0 > x_{eq\_unstable} > x_{eq\_stable} \geq 0$ . In addition, in a vibrating environment, when the mobile mass oscillates, it must never go out of the attraction zone of the stable equilibrium point. This attraction zone is delimited by  $x_{eq\_unstable}$  which corresponds to the maximal allowed displacement  $x_{max}$  and to the maximal transducer capacitance  $C_{max}$  [4].

The stable and unstable positions of the mobile electrode in OPGC device are given by resolving the equation:

$$x(d_0 - x)^2 = \frac{\varepsilon S U_0^2}{2k} \quad (2)$$

For the IPGC architecture, the stable position is  $x = 0$  and the unstable position of the mobile mass is given by:

$$x_{unstable} = \pm \sqrt{d_0^2 - U_0 \sqrt{\frac{2\varepsilon N S d_0}{k}}} \quad (3)$$

where  $N$  is the number of fingers attached to the mobile mass and  $S$  the overlapped surface between two fingers. The variable capacitance is at its minimal value when the mobile mass is at its rest position.

In [4] we assumed, in order to determine  $C_{min}$ , that for OPGC VEH the mass oscillates symmetrically around the static stable equilibrium point. However for a more accurate value of  $C_{min}$  we have to take into account the influence of the electrostatic force when the electrodes are close. A first approximation consists in assuming a symmetrical displacement around the position  $x_{med}$  corresponding to the half of the maximal electrostatic force:

$$x_{med} = \frac{F_{elec\_max}}{2k} = \frac{1}{4} U_0^2 \frac{\varepsilon S}{(d_0 - x_{unstable})^2} \quad (4)$$

The position of the mobile electrode corresponding to the minimal capacitance  $C_{min}$  is then given by:  $x_{min} = 2x_{med} - x_{max}$ .

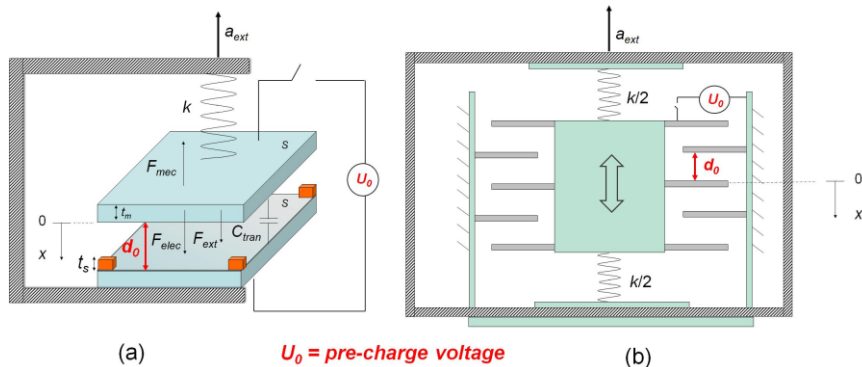


Fig. 1. Architectures of the OPGC (a) and IPGC (b) Vibration Energy Harvesters.

### 3. Optimization of the design

The design optimization consists in calculating  $\{d_0, U_0\}$  such as the harvested power  $P'_{h\_max}$  is maximised and the voltage across the transducer  $U_{Ctran}$  is lower than the maximal voltage allowed by the system. In [4], we showed that in order to get a maximum of converted power, it is better to work with a low pre-charge voltage allowing a large  $C_{max}/C_{min}$  ratio. Then the main limitation comes from the voltage  $U_{Ctran\_max}$  allowed by the conditioning electronic, which is equal to  $C_{max}/C_{min}$  times  $U_0$ .

#### 3.1. Consequence of a voltage limitation on $C_{tran}$

In Fig. 2, we trace the evolution of  $P'_{h\_max}$  as a function of  $U_{Ctran\_max}$  for various values of  $d_0$ . The first part of each curve (in dotted line) is associated to the lower values of  $U_0$  and corresponds to  $x_{eq\_unstable}$  beyond the stoppers, which is not possible.  $C_{max}/C_{min}$  remains constant at its maximum value corresponding to  $x_{max} = d_0 - t_{s\_min}$  and  $P'_{h\_max}$  increases proportionally to  $U_0^2$ . The top of curves corresponds to the optimum pre-charge when the mobile electrode is just in contact with the stoppers. From this point, any increase of  $U_0$  decreases  $P'_{h\_max}$  since  $x_{eq\_unstable}$  and then  $C_{max}/C_{min}$  decrease. If a voltage limitation is set, this extremum has to fit with it in order to maximize the harvested power. For instance, for our OPGC device, the best design is with a gap  $d_0 = 26.5 \mu\text{m}$  and a pre-charge of 2.91 V, leading to a maximal harvested power of 14.9  $\mu\text{W}$ . In these conditions, the  $C_{max}/C_{min}$  ratio is 895 pF / 43 pF  $\sim 20$ .

For the IPGC architecture, the best design is with a gap  $d_0 = 46.7 \mu\text{m}$  and a pre-charge of 3.15 V, leading to a maximal harvested power of 33.6  $\mu\text{W}$ . The  $C_{max}/C_{min}$  ratio is 940 pF / 50 pF  $\sim 19$ .

#### 3.2. Consequences of a variation of the external acceleration

If a reduction of 10% of the mobile plate's displacement occurs on our previous optimized OPGC transducer, the  $C_{max}/C_{min}$  ratio decreases to 259 pF / 29 pF  $\sim 9$ . If keeping the same  $U_0$  and  $d_0$ , the new maximal harvested power dramatically decreases to 1.77  $\mu\text{W}$  (Fig. 2). However, the maximal harvested power would be optimized for a pre-charge voltage  $U_0 \sim 10 \text{ V}$  and would be equal to 12.3  $\mu\text{W}$ . In these conditions,  $C_{max}/C_{min}$  is  $\sim 5$ .

Therefore two approaches are possible in order to limit the impact of the acceleration's amplitude variations. We can have a smart system where the pre-charge can be adjusted to the fluctuation of the acceleration during the conversion process (such architecture has been proposed by A. Dudka et al in [5]), or we have to design a system much less sensitive. This can be obtained with larger stoppers leading to a smaller value of  $C_{max}$  (and then a smaller  $C_{max}/C_{min}$  ratio).

### 4. VHDL-AMS modeling

We have validated our results with a VHDL-AMS/ELDO modeling and performed a simulation of the OPGC transducer implemented in the circuit proposed by Miranda et al. [1].

The Fig. 3 presents the behavior of the optimized transducer for a  $C_{max}/C_{min}$  ratio of 4. The graph shows the

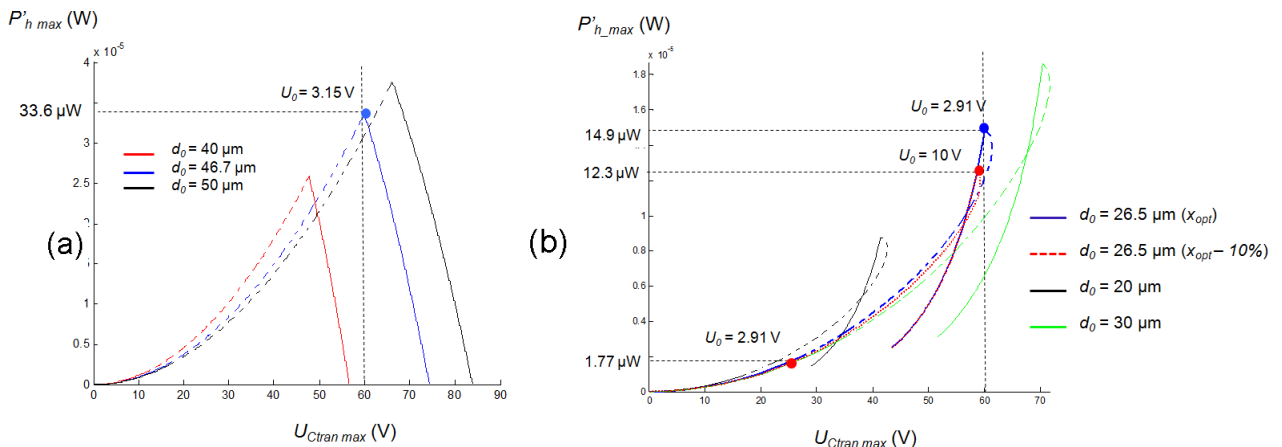


Fig. 2. Evolution of the maximal harvested power  $P'_{h\_max}$  as a function of the maximal voltage across the transducer  $U_{Ctran\_max}$  for the IPGC (a) and the OPGC (b) architectures.

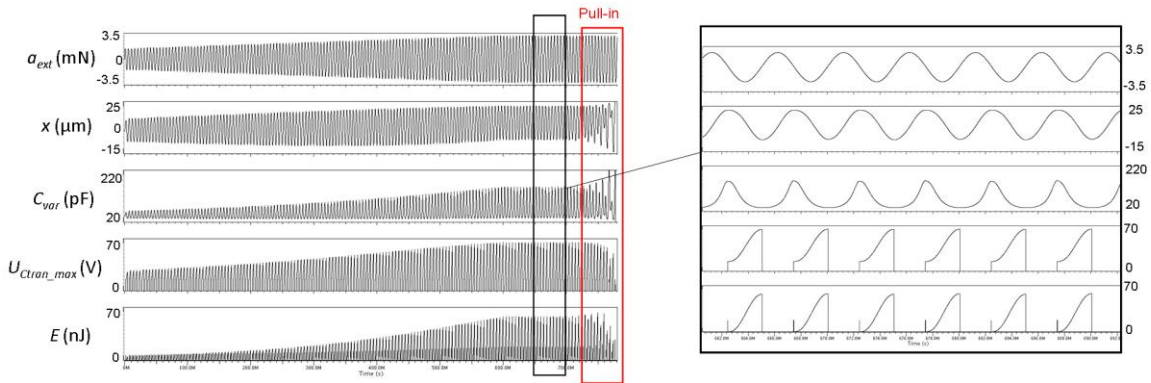


Fig. 3. VHDL-AMS modeling of the optimized OPGC device with a  $C_{max}/C_{min}$  ratio of 4 and pre-charged with  $U_0 = 15$  V

displacement  $x$  of the mobile electrode when it's submitted to an external sinusoidal acceleration  $a_{ext}$  at the device mechanical resonance. The graph shows also the variations of  $C_{var}$ ,  $U_{Cvar\_max}$  and the energy  $E$  harvested by the transducer per capacitance variation cycle.

We observe a shift of the median position  $x_{med}$  which is due to the shift of the resonance frequency of the resonator when high electrostatic coupling with the spring-mass system occurs. The amplitude of the oscillations,  $U_{Cvar\_max}$  and the harvested energy increase naturally with the external acceleration. As the stoppers are not taking into account, the pull-in occurs at the end of the simulation.

When  $a_{ext}$  is large enough to allow a maximal displacement of the electrode,  $U_{Cvar\_max}$  is saturating at  $\sim 60$  V. The harvested energy is about 57 nJ so the maximal harvested power is  $57 \text{ nJ} \times 200 \text{ Hz} = 11.4 \text{ } \mu\text{W}$  as predicted in our calculations. If the external acceleration is reduced of 10 %, the new maximal harvested power is about  $8.3 \text{ } \mu\text{W}$ . This result matches our calculations.

## 5. Conclusion

We have detailed how to design the OPGC and IPGC VEH in order to harvest the maximum of power. The originality of this work consists in taking into account both electrical and mechanical aspects for the transducer's design optimisation and a constraint on the maximal voltage allowed across the transducer's terminal. We studied the case where the vibration's characteristics are known and also the influence of 10 % decrease of the mobile electrode's displacement. It appears that the decrease of the external acceleration provides a dramatically fall about 90 % of the maximal harvested power, for high  $C_{max}/C_{min}$  ratio. If the voltage  $U_0$  is adjusted, the decrease is only about 20 %. So, it would be very useful to have an adaptive system as the one described in [5]. For a passive device, a much less sensitive system has to be designed, i.e. with a smallest  $C_{max}/C_{min}$  ratio, for example with  $C_{max}/C_{min} = 4$ . Then, the same decrease of the external acceleration will induce a power loss of only 30 %. Our results have been validated with a behavioral VHDL-AMS modeling of the OPGC transducer implemented in the conditioning circuit of Miranda [1].

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