Morphological Analysis of Point Sets :

Application to Digital Histopathology

Université Paris Descartes
UFR de Mathématiques et Informatique
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June 6th, 2013

Outline

- Motivation ;
- Point Set Morpho. Math. ;
- Spatial Reasoning ;
- Around Digital Histopathology.

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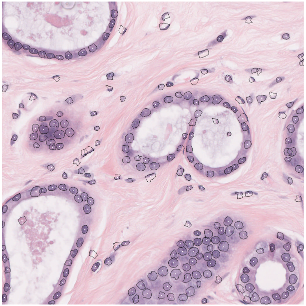
Digital Histopathology



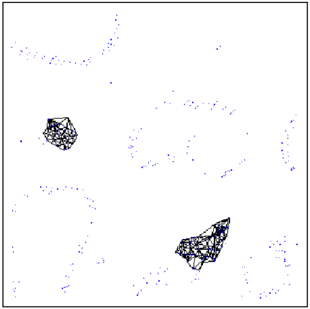
- An ongoing big challenge : academic, industrial, societal ;
- A complete ground test : closed universe, no digital model, possibility of ground truth ;
- A new avenue to put together semantic filtering and image processing ;

Pathology Innovation Centre of Excellence (PICOE). Digital Histopathology : A New Frontier in Canadian Healthcare. White Paper. Jan. 2012. GE Healthcare.

Point Sets

- A methodological new trend : point reconstruction, sparse representation, points of interest, PCL library ;
- A more geometric approach to image analysis issues : to overcome radiometric instability and redundancy ;
- A re-energized theoretical topic : visual point set processing ;

Mathematical Morphology



- A well-established theoretical framework : lattice theory (Gallois, abduction, Minkovsky operators etc.)
- A global concept : image processing and reasoning as well ;
- A new theoretical topic : morphology and spatial relations over point sets (ANR DESCRIBE 2012-2014) ;

Collaborators

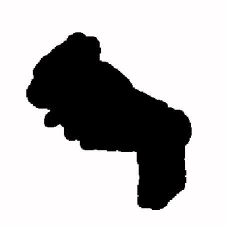
	IPAL, Singapore
	I2R, A*STAR, Singapore
	NUS, Singapore
	Pitié-Salpêtrière Hospital, France

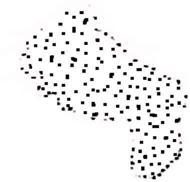
Outline

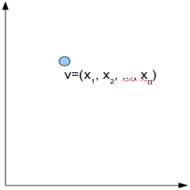
- Motivation ;
- Point Set Morpho. Math. ;
- Spatial Reasoning ;
- Around Digital Histopathology.

Shape Definition

Shape ?







Shape as :(a) a segmented subset of $\Re^2$;

(b) a point set representation S ;

(c) a feature vector $v \in \Re^n$.

Edelsbrunner, H., & Kirkpatrick, D.G. : On the shape of set of points in the plane. IEEE Trans. Inform. Theory, **29** *:551-559. (1983)*

Shape Definition

Stereoscopic data

- Shape ;
- Silhouette ;
- Cluster.

Lomenie, N., Gallo, L., Cambou, N., & Stamon, G. Morphological Operations on Delaunay Triangulations. ICPR *:556-559 (2000).*

Shape Definition

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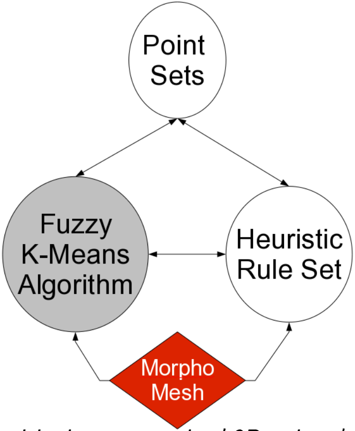

Lomenie, N. A generic methodology for partitioning unorganized 3D point clouds for robotic vision. Canadian Conference on Computer and Robot Vision *:172-181 (2004)*

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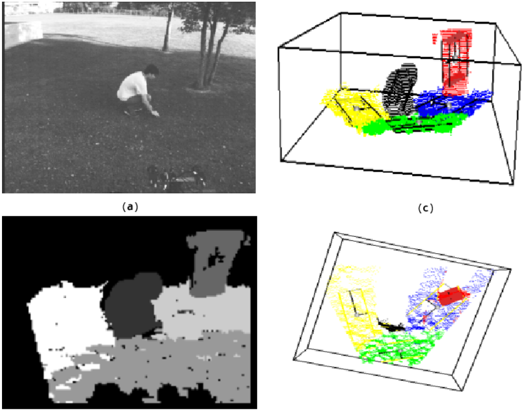

(a) (c) (b) (d)

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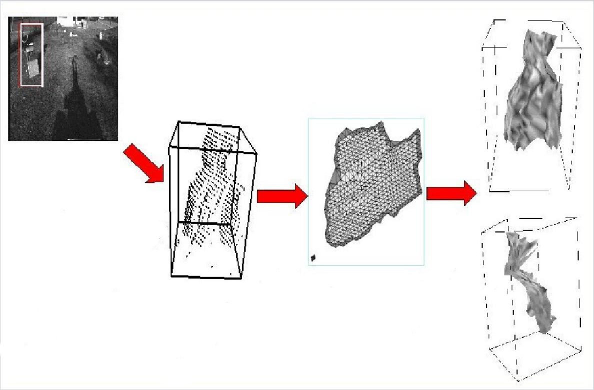

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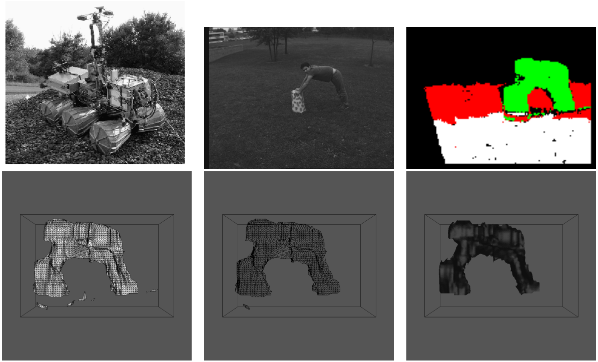
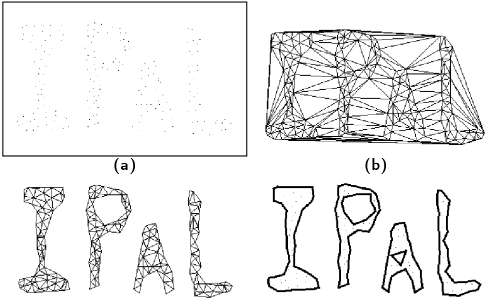


Shape Definition

α-objects

- Topological context ;
- Graph context ;
- Algorithmic context.

(a) 2D point set S ; (b) Delaunay triangulation $Del(S)$ or ∞-complex(S) ; (c) α_{opt}-complex (as a simplicial complex) $C_{\alpha_{opt}}(S)$ triangulated by $Del_{\alpha_{opt}}(S)$; (d) α_{opt}-shape as a polytope $S_{\alpha_{opt}}(S)$



(a) (b) (c) (d)

$S_\infty = conv(S)$, *where conv stands for the convex hull and* $S_0 = S$

Lomenie, N. & Stamon, G. Point Set Analysis, Advances in Imaging and Electron Physics, Peter W. Hawkes, San Diego : Academic Press, vol. 167, pp. 255-294 (2011)

Shape Definition

α-objects

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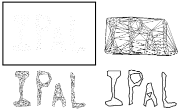

α-**ball.** For $0 < \alpha < \infty$, let an α-ball B_α be an open ball of $\Re^2$ with radius α.

B_0 is a point and B_∞ is an open half-space.

B_α is empty if $B_\alpha \cap S = \emptyset$.

Such an α-ball is denoted $B_\alpha^\emptyset$.

k-**simplex.** $\sigma_T = conv(T)$, $T \subseteq S$ and $|T| = k + 1$ for $0 \leq k \leq 2$.

H. Edelsbrunner, E.P. Mucke, E.P., Three-dimensionnal alpha-shapes, ACM Transactions on Graphics 13 (1), pp. 43-72 (1994)

Shape Definition

α-objects

- Topological context ;
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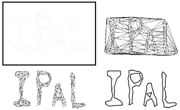

k-faces and S_α. For all k-simplex σ_T, $0 \leq k \leq 1$,

$$\sigma_T \text{ is } \alpha - \text{exposed} \iff \exists B_\alpha^\emptyset \ / \ T = \partial B_\alpha^\emptyset \cap S$$

A fixed α defines sets $F_{k,\alpha}$ of α-exposed k-simplices for $0 \leq k \leq 1$.

The α-shape S_α of S is the polytope whose boundary consists of the edges in $F_{1,\alpha}$ and the vertices in $F_{0,\alpha}$:

$$\partial S_\alpha = \bigcup_{0 \leq k \leq 1} F_{k,\alpha}$$

The k-simplices in $F_{k,\alpha}$ are also called the k-faces of S_α.

Shape Definition

α-objects

- Topological context ;
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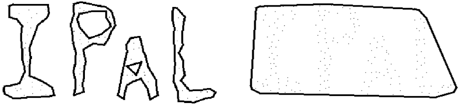

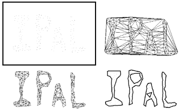

α-**hull.** We can define related geometrical structures such as the α-convex hull H_α of S :

$$H_\alpha(S) = \{\bigcup B_\alpha^\emptyset\}^C$$

Then we have the following property :

$$H_\infty(S) = conv(S) = S_\infty(S)$$

Shape Definition

α-objects

- Topological context ;
- Graph context ;
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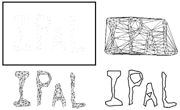

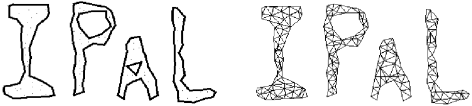

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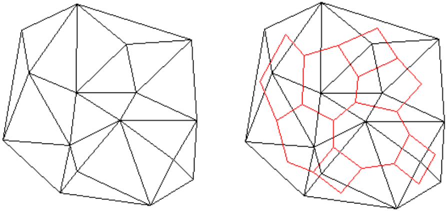
Shape Definition

α-objects and Voronoi Graph

- Topological context ;
- Graph context ;
- Algorithmic context.

Delaunay triangulation. A Delaunay triangulation for a set S of points in a plane is a triangular mesh $Del(S)$ such that no point in S is inside the circumcircle of any triangle in $Del(S)$. $(\forall \sigma_T^2, B_{\sigma_T} \cap S = \emptyset)$



Shape Definition

α-objects and Voronoi Graph

- Topological context ;
- Graph context ;
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Del **and** S_α. By definition, for each k-simplex σ_T *in Del*, there exists values of α so that σ_T is $\alpha - exposed$.
Conversely, every face of S_α is a simplex of *Del*.
This implies the relationship between the Delaunay triangulation and the boundary of S_α :

$$For\ 0 \leq k \leq 1, F_k = \bigcup_{0 \leq \alpha \leq \infty} F_{k,\alpha}$$

$$Del(S) = \bigcup_{0 \leq k \leq 1} F_k = \bigcup_{0 \leq \alpha \leq \infty} \partial S_\alpha.$$

Shape Definition

α-objects and Voronoi Graph

- Topological context ;
- Graph context ;
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Simplicial Complex (1). A simplicial complex is a topological space of a certain kind, constructed by "gluing together" points, line segments, triangles, and their n-dimensional counterparts.

Simplicial Complex (2). A simplicial complex $\mathcal{K}$ is a set of k-simplices σ_T that satisfies the following conditions :

1. Any face of a simplex from $\mathcal{K}$ is also in $\mathcal{K}$.
2. The intersection of any two simplices $\sigma_1, \sigma_2 \in \mathcal{K}$ is a face of both σ_1 and σ_2.

Shape Definition

α-objects and Voronoi Graph

- Topological context ;
- Graph context ;
- Algorithmic context.

α-**complex.** As a simplicial subcomplex.

$$C_\alpha(S) = \{\sigma_T \in Del / \sigma_T \in \bigcup_{0 \leq k \leq 2} G_{k,\alpha} \text{ or } \sigma_T \in \partial \sigma_T^{k+1} \text{ with } \sigma_T^{k+1} \in C_\alpha\}$$

with

$$G_{k,\alpha} = \{\sigma_T \in Del / B_T \text{ empty and } \rho_T < \alpha\} \text{ and } \forall \alpha, G_{0,\alpha} = S$$

Property. $\forall 0 \leq \alpha \leq \infty, S_\alpha = |C_\alpha|$

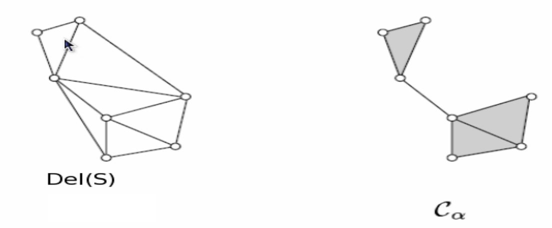
Shape Definition

α-objects and Voronoi Graph

- Topological context ;
- Graph context ;
- Algorithmic context.

As $C_\alpha(S) \subseteq C_\infty(S) = Del(S)$, we can define α-**Delaunay triangulation.**

$$Del_\alpha(S) = \{\sigma_T \in Del / \sigma_T \in G_{2,\alpha} \text{ or } \sigma_T \in \partial\sigma_T^{k+1} \text{ with } \sigma_T^{k+1} \in Del_\alpha\}$$



$$\forall 0 \leq \alpha \leq \infty, S_\alpha = |C_\alpha| = |Del_\alpha|$$

Shape Definition

Obtaining α-objects

- Topological context ;
- Graph context ;
- Algorithmic context.

For each σ_T, two values λ_T and μ_T are derived :

$$\begin{cases} \textit{if } |T| = 3, & \lambda_T = \mu_T = \rho_T \\ \textit{else} & \begin{cases} \lambda_T = \textit{min}\,\{\lambda_{T'}\,|\sigma_{T'} \in \textit{up}(\sigma_T)\,\} \\ \text{and} \\ \mu_T = \textit{max}\,\{\mu_{T'}\,|\sigma_{T'} \in \textit{up}(\sigma_T)\,\} \end{cases} \end{cases}$$

$$\textit{up}(\sigma_T) = \{\sigma_{T'} \in \textit{Del} \mid T \subset T' \text{ and } |T'| = |T| + 1\}$$

Shape Definition

Obtaining α-objects

- Topological context ;
- Graph context ;
- Algorithmic context.

σ_T is...	*Singular*	*Regular*	*Interior*
Triangle			$\alpha \in [\rho_T, \infty[$
Edge,			
$\notin \partial conv(S)$	$\alpha \in [\rho_T, \lambda_T[$	$\alpha \in [\lambda_T, \mu_T[$	$\alpha \in [\mu_T, \infty[$
$\in \partial conv(S)$	$\alpha \in [\rho_T, \lambda_T[$	$\alpha \in [\lambda_T, \infty[$	
Vertex,			
$\notin \partial conv(S)$	$\alpha \in [0, \lambda_T[$	$\alpha \in [\lambda_T, \mu_T[$	$\alpha \in [\mu_T, \infty[$
$\in \partial conv(S)$	$\alpha \in [0, \lambda_T[$	$\alpha \in [\lambda_T, \infty[$	

$$C_\alpha = \{\text{Singular } \sigma_T\} \cup \{\text{Regular } \sigma_T\} \cup \{\text{Interior } \sigma_T\}$$

$$Del_\alpha = \{\text{Interior } \sigma_T\}$$

$$\partial S_\alpha = \{\text{Regular } \sigma_T\}$$

Shape Definition

Obtaining α-objects

- Topological context ;
- Graph context ;
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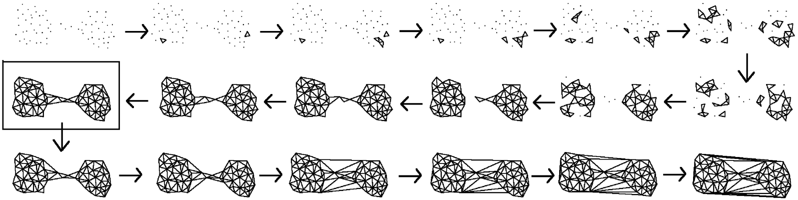
$$C_\alpha = \{\text{Singular } \sigma_T\} \cup \{\text{Regular } \sigma_T\} \cup \{\text{Interior } \sigma_T\}$$
$$Del_\alpha = \{\text{Interior } \sigma_T\}$$
$$\partial S_\alpha = \{\text{Regular } \sigma_T\}$$

α-objects

Binarization of a Point Set S



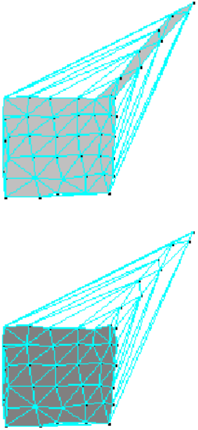
A spectrum of $\alpha -$ *objects* derived from the Edelsbruner's modeling : from S_0 to the meshed convex hull S_∞ of S.

$$\forall S, \alpha\text{-bin}(S) = \{T \in Del(S) \mid \rho_T < \alpha\} \equiv Del_\alpha(S)$$

$$|\alpha\text{-bin}(S)| = |Del_\alpha(S)| = |S_\alpha(S)| \textit{ and } \alpha_{opt} = 2 * \text{median}_{T \in Del(S)}(\alpha)$$

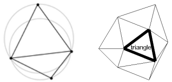
Shape Analysis

A Morphological Study



In the α-object framework :

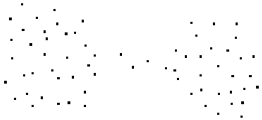
$$\forall k \in N, e_T^k = max\{e_{T'}^{k-1} | T' \in neighbor(T)\}$$
$$\forall k \in N, d_T^k = min\{d_{T'}^{k-1} | T' \in neighbor(T)\}$$
$$and \ e_T^0 = d_T^0 = \rho_T$$



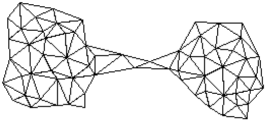
$$neighbor(T) = \{T' \in Del | T' \cap T \neq \emptyset \ and \ |T'| = |T| = 3\}$$

Shape Analysis

Eroding S



(a) A point set S in $\Re^2$;



(b) Its binarized representation ;

$$\forall\ T, e_T = max\{\rho_T | T' \in neighbor(T)\}$$
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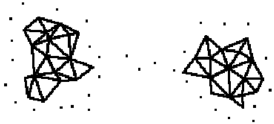
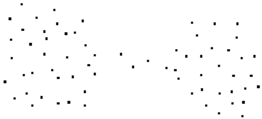


(c) α-eroded(S) for $\alpha = \alpha_{opt}$

$$\alpha\text{-eroded}(S) = \{T' \in Del | e_{T'} < \alpha\}$$

Shape Analysis

Dilating S



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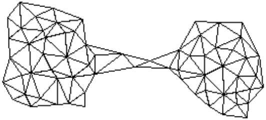

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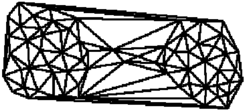


(c) α-dilated(S) for $\alpha = \alpha_{opt}$

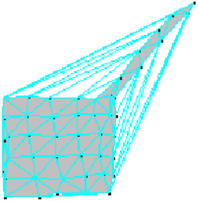
$$\forall\ T, e_T = max\{\rho_T | T' \in neighbor(T)\}$$

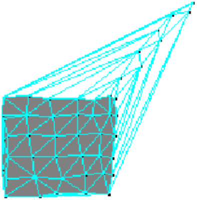
$$\forall\ T, d_T = min\{\rho_T | T' \in neighbor(T)\}$$

$$\alpha\text{-dilated}(S) = \{T' \in Del | d_{T'} < \alpha\}$$

Shape Analysis

A Morphological Study





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$$and\ e_T^0 = d_T^0 = \rho_T$$

$$neighbor(T) = \{T' \in Del | T' \cap T \neq \emptyset \ and\ |T'| = |T| = 3\}$$

Shape Analysis

Playing with the new α-objects(S)

k acting as the size of the structuring element

$$\alpha^k\text{-eroded}(S) = \{T' \in Del | e^k_{T'} < \alpha \text{ and } |T'| = 3\}$$

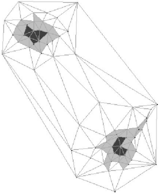
$$\alpha^k\text{-dilated}(S) = \{T' \in Del | d^k_{T'} < \alpha \text{ and } |T'| = 3\}$$

$$(\alpha^k\text{-dilated}(S))^C = \{T' \in Del | e^k_{T'} > \alpha \text{ and } |T'| = 3\}$$

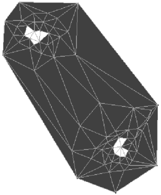
Shape Analysis

Playing with the new α-objects(S)

Duality of the transformations : (a) In gray, the α-complex and in black, the α-eroded complex (b) In black, the α-dilated of the complementary α-complex



(a)

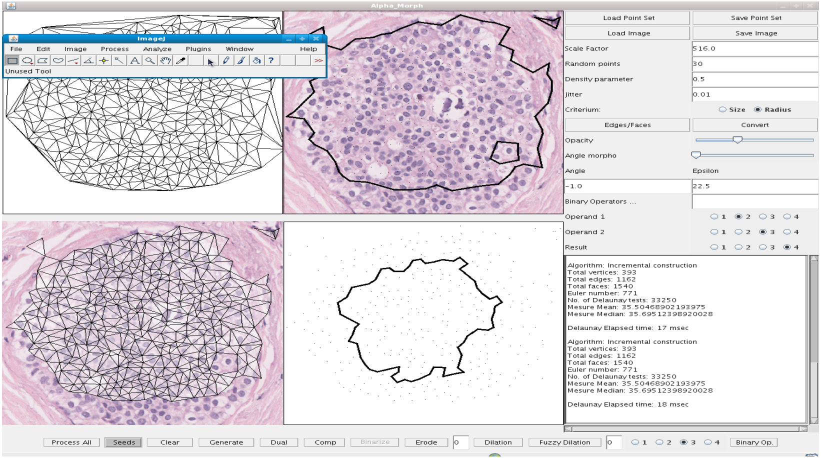


(b)

MorphoMesh Package - Java ImageJ plugin : ***http://www.math-info.univ-paris5.fr/~lomn/Data/MorphoMesh.zip***

Shape Analysis

Playing with the new α-objects(S)



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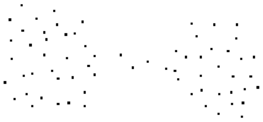
Shape Analysis

Opening and Closing S



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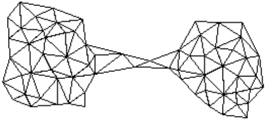
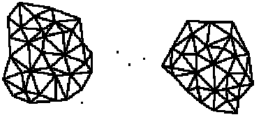


(b) Its binarized representation ;



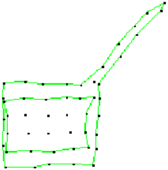
(c) α-open(S) for $\alpha = \alpha_{opt}$

$$
\begin{aligned}
o_T^k &= min\{e_{T'}^k \,|\, T' \in neigh(T)\} \\
&= min_{T' \in neigh(T)}\{max_{T'' \in neigh(T')}\{e_{T''}^{k-1}\}\}
\end{aligned}
$$

$$\alpha^k\text{-opened} = \{T \in Del | o_T^k < \alpha \; and \; |T| = 3\}$$

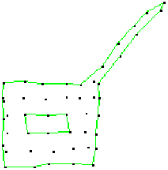
Shape Analysis

Duality Edge/Face



$$\lambda_e^k = min\{e_T^k \ , \ e_{T'}^k\} \quad and \quad \mu_e^k = max\{e_T^k \ , \ e_{T'}^k\}$$

$$\lambda_o^k = min\{o_T^k \ , \ o_{T'}^k\} \quad and \quad \mu_o^k = max\{o_T^k \ , \ o_{T'}^k\}$$



Successive Erosions in Contour Mode

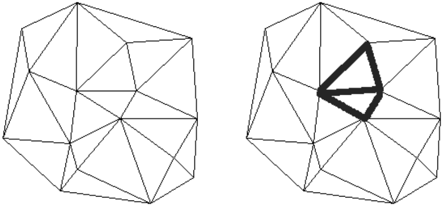
Morph. operator	*Region Mode*	*Contour Mode*
$\alpha^k - eroded$	$\alpha \in [e^k, \infty[$	$\alpha \in [\lambda_e^k, \mu_e^k]$
$\alpha^k - opened$	$\alpha \in [o^k, \infty[$	$\alpha \in [\lambda_o^k, \mu_o^k]$

N.B. : $\rho \rightarrow \phi = \frac{1}{\rho} \longrightarrow \min \rho \rightarrow \max \phi$ *and* $\alpha \rightarrow \frac{1}{\alpha}$

Lattice Framework for Point Sets

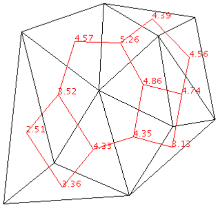
To any point set S is associated :

- Its Delaunay triangulation $Del(S)$;
- The set $\wp(Del)$ of all the corresponding sub-triangulations D_i of Del.



Lattice Framework for Point Sets

For any point set S, two complete lattice structures are defined :



- Within the set theory frame, called $\mathcal{L}_1 = (\wp(Del), \subseteq)$ where $D_1 \subseteq D_2$ denotes the relation : $\forall T \in Del, T \in D_1 \rightarrow T \in D_2$;
- Within the functional theory frame, called $\mathcal{L}_2 = (\mathcal{M}(Del), \leq)$, where $\mathcal{M}(Del)$ is the set of meshes on Del, i.e., the set of mappings from the triangles T in Del to ρ_T values, and where the partial ordering $\leq$ is defined by : $\forall M_1 \text{ and } M_2 \in \mathcal{M}(Del),\ M_1 \leq M_2 \iff \forall T \in Del, \rho_T^1 \geq \rho_T^2$

Lattice Framework for Point Sets

As for $\mathcal{L}_1$,

$$\forall D_1, D_2 \in \mathcal{L}_1,\ \inf(D_1, D_2) = D_1 \cap D_2$$

As for $\mathcal{L}_2$,

$$\forall M_1, M_2 \in \mathcal{L}_2,\ \inf(M_1, M_2) = \{T \in Del, max(\rho_T^1, \rho_T^2)\}$$

Similarly, the *supremum* operators are given by :

$$\forall D_1, D_2 \in \mathcal{L}_1,\ \sup(D_1, D_2) = D_1 \cup D_2$$

and

$$\forall M_1, M_2 \in \mathcal{L}_2,\ \sup(M_1, M_2) = \{T \in Del, min(\rho_T^1, \rho_T^2)\}$$

Lattice Framework for Point Sets

Binarization of a Mesh M

Relation between the mesh M and the point set S :

$$\forall M \in \mathcal{M}(Del(S)),\ \alpha\text{-bin}(M) = \{\, T \in Del(S) \mid \rho_T < \alpha\} \equiv Del_\alpha(S)$$

$$|\alpha\text{-bin(M)}| = |\alpha\text{-bin(S)}| = |Del_\alpha(S)| = |S_\alpha(S)|$$

Lattice Framework for Point Sets

Erosion and Dilation

$$\forall M \in \mathcal{M}(Del(S)),$$
$$\mathrm{e}(M) = \{T \in Del, e_T\}$$
$$\mathrm{d}(M) = \{T \in Del, d_T\}$$

$$\forall\ T, e_T = max\{\rho_T | T' \in neighbor(T)\}$$
$$\forall\ T, d_T = min\{\rho_T | T' \in neighbor(T)\}$$

$$\forall M \in \mathcal{M}(Del(S)),$$
$$\alpha\text{-bin}(e^{\mathcal{L}}(M)) \equiv \alpha\text{-eroded}(S)$$
$$\alpha\text{-bin}(d^{\mathcal{L}}(M)) \equiv \alpha\text{-dilated}(S)$$

Lattice Framework for Point Sets

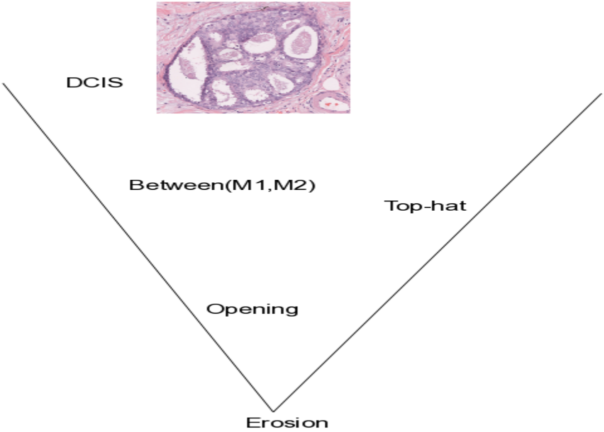
Opening and Closing S

$$\forall M \in \mathcal{M}(Del(S)),\ \mathrm{o}(M) = \mathrm{d} \circ \mathrm{e}(M) \text{ and } \mathrm{c}(M) = \mathrm{e} \circ \mathrm{d}(M)$$

$$\forall M \in \mathcal{M}(Del(S)),\ \mathrm{o}^n(M) = \mathrm{d}^n \circ \mathrm{e}^n(M)$$

$$\forall n > 1,\ \mathrm{o}^n(M) \neq \mathrm{o}(M)\ (N.B.\, \mathrm{o}^2(M) \neq \mathrm{o} \circ \mathrm{o}(M))$$

Structural Operators on S

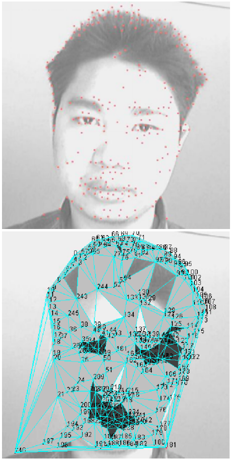


N. Lomenie & G. Stamon : Morphological Mesh filtering and alpha-objects, Pattern Recognition Letters, **29***(10) :1571-1579. (2008)*

Outline

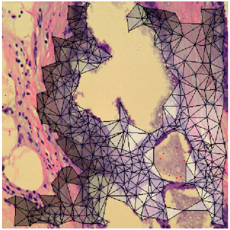
- Motivation ;
- Point Set Morpho. Math. ;
- Spatial Reasoning ;
- Discussion around Digital Histopathology.

Spatial Reasoning



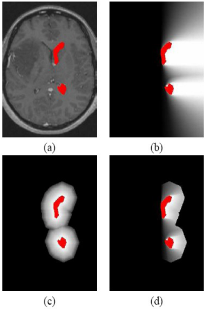
- Topological/Mathematical Analysis vs. Linguistic/Structural Representation ;
- A bunch of operators and applications based on the mathematical morphology framework (abduction etc.) ;
- e.g. Spatial Relations.

Spatial Reasoning



- Topological/Mathematical Analysis vs. Linguistic/Structural Representation ;
- A bunch of operators and applications based on the mathematical morphology framework (abduction etc.) ;
- e.g. Spatial Relations.

Spatial Relationships
and Mathematical Morphology



- Which is the region of space corresponding to a spatial query about a reference object M_i ? And, if necessary, what is the fuzzy mesh description of this region ?
- To which degree an object O belongs to that region ?
- Fuzzy representation and spatial reasoning made possible.

I. Bloch, O. Colliot, R.M. Cesar, On the ternary spatial relation "between", IEEE Trans. on Systems, Man, and Cybernetics, Part B : Cybernetics 36 (2) (2006) 312-327.

Spatial Relationships

The 'between' relation

$$\beta_{dil}(M_1, M_2) = d^n[d^n(M_1) \cap d^n(M_2)] \cap M_1^C \cap M_2^C$$
$$with \quad n = inf\{k / d^k(M_1) \cap d^k(M_2) \neq \emptyset\}$$

$$\beta^1_\alpha(M_1, M_2) = d^n_\alpha(M_1) \cap d^n_{\pi+\alpha}(M_2) \cap M_1^C \cap M_2^C$$

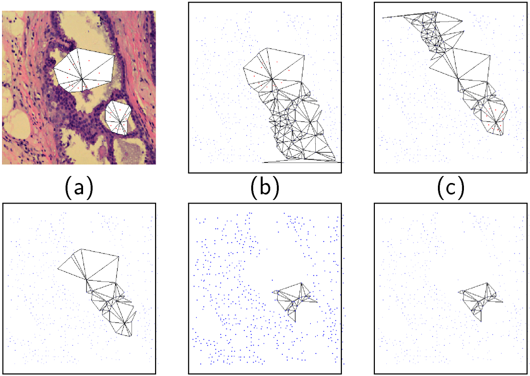
$$\beta^2_\alpha(M_1, M_2) = d_\alpha(M_1) \cap d_{\pi+\alpha}(M_2) \cap M_1^C \cap M_2^C$$
$$\cap [d_\alpha(M_1) \cap d_\alpha(M_2)]^C \cap [d_{\pi+\alpha}(M_1) \cap d_{\pi+\alpha}(M_2)]^C$$

Loménie, N. and Racoceanu, D. (2012). Point set morphological filtering and semantic spatial configuration modeling : application to microscopic image and bio-structure analysis, Pattern Recognition, **45***(8) :2894-2911*

Spatial Relationships

The 'between' relation

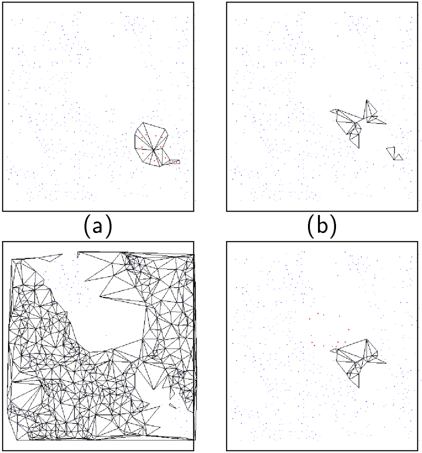
(a) Two sub-meshes M_1 and M_2 ;
The dilated meshes at order n (b) $d^n_\alpha(M_1)$ and (c) $d^n_{\pi+\alpha}(M_2)$;
(d) The intersection $M_1^C \cap M_2^C$;
(e) The region between $\beta^1_\alpha(M_1, M_2)$;
(f) after an isotropic opening $o(\beta^1_\alpha(M_1, M_2))$.


(a) (b) (c) (d) (e) (f)

Spatial Relationships

The 'between' relation

(a) One non convex sub-mesh of interest M_2 ; (b) Result with the second definition $\beta^1_\alpha(M_1, M_2)$; (c) $[d_\alpha(M_1) \cap d_\alpha(M_2)]^C \cap [d_{\pi+\alpha}(M_1) \cap d_{\pi+\alpha}(M_2)]^C$; (d) Result with the third definition $\beta^2_\alpha(M_1, M_2)$.



(a) (b) (c) (d)

Spatial Relationships

Fuzzy Representations

Algorithm 1: Algorithmic definition of the fuzzy dilatation d_ν^f

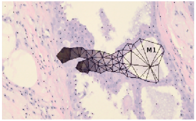
Data: A mesh
$M = T \in Del(S), \phi_T \in [0,1]$
defined over the lattice $\mathcal{L}$

Result: A resulting mesh $d^f(M)$

for $i \leftarrow 0$ **to** $N+1$ **do**
  foreach $T \in Del$ **do** $d_T = 0$;
  $d_T = \max\{d_T, \max_{T' \in \nu(T)} \{(\phi_T, \phi_{T'} + (1 - i/N) - 1\}\}$;
  foreach $T \in Del$ **do** $\phi_T = d_T$;

end

$$\forall M \in \mathcal{M}(Del(S)),$$
$$M = \{T \in Del(S), \phi_T \in [0,1]\},$$
$$M^c = \{T \in Del, 1 - \phi_T\}$$



(a) Fuzzy landscape for the "Left region" of M_1 ;

Spatial Relationships

Fuzzy Representations

Algorithm 2: Algorithmic definition of the fuzzy dilatation d_ν^f

Data: A mesh
$M = T \in Del(S), \phi_T \in [0,1]$
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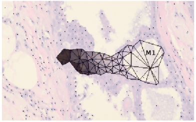
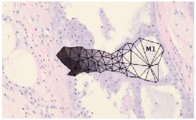
  $d_T = \max\{d_T, \max_{T' \in \nu(T)} \{(\phi_T, \phi_{T'} + (1 - i/N) - 1\}\}$;
  foreach $T \in Del$ **do** $\phi_T = d_T$;
end

(b) Post-processed mesh after an isotropic opening ;
(c)The *Near and Left* region :
$\text{Around}_{FuzDil}(M_1) * o_{\nu_{iso}}(\text{Left}_{Dil}(M_1))$.



(b)

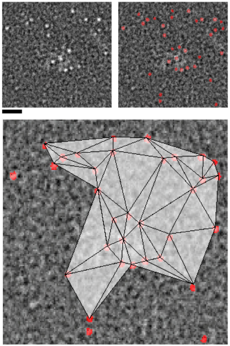


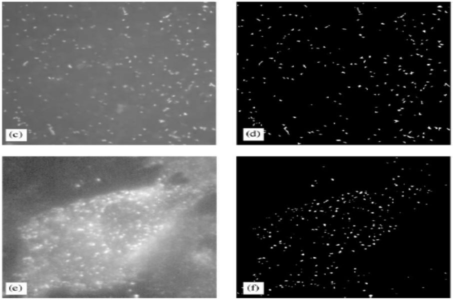
(c)

Loménie, N. and Racoceanu, D. (2012). Point set morphological filtering and semantic spatial configuration modeling : application to microscopic image and bio-structure analysis, *Pattern Recognition*, **45**(8) :2894-2911

Spatial Reasoning

Computational Topology





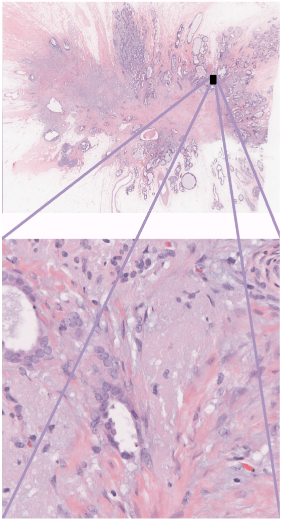
Kyoshitaka Kimori1, Norio Baba, Nobuhiro Morone. Extended morphological processing : a practical method for automatic spot detection of biological markers from microscopic images. BMC Bioinformatics, **11** *:373, (2010)*

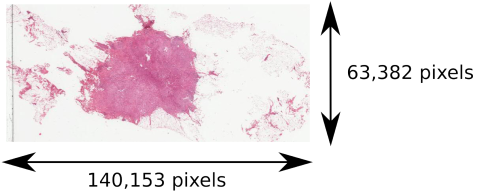
Olivo-Marin J-C : Extraction of spots in biological images using multiscale products. Pattern Recognition, **35** *:1989-1996 (2002).*

Discussion Overview

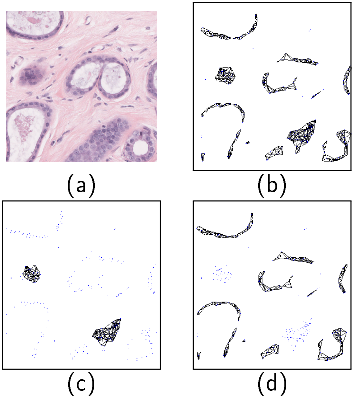
- Motivation ;
- Point Set Morpho. Math. ;
- Spatial Reasoning ;
- Discussion around Digital Histopathology.

Digital Histopathology



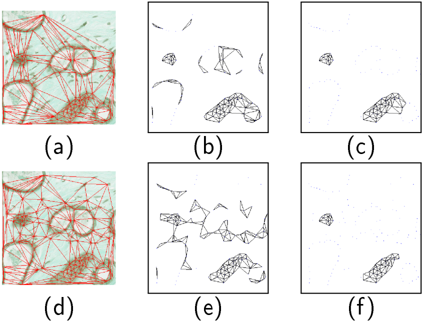


Digital Histopathology



(a) A 1024 x 1024 pixels sub-image out of a WSI ; (b) $Del_\alpha(S)$; (c) Focus on Ductal Carcinoma In Situ (DCIS)areas with $o^2(Del_\alpha(S))$; (d) Focus on normal cells with $Del_\alpha(S) \cap (o^2(Del_\alpha(S)) \cap Del_\alpha(S))^c$

Digital Histopathology

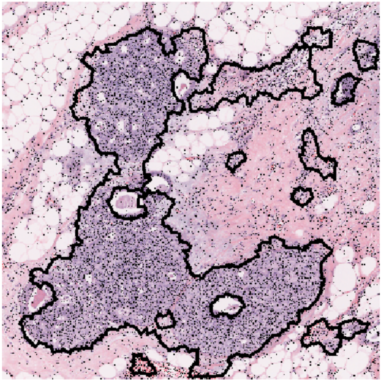


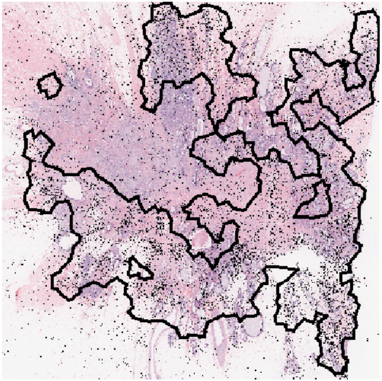
Point set (a) without artifacts M ; (d) and with artifacts M_b ; (b) Binarization of M ; (e) Binarization de M_b ; (c) Opening of M ; (f) (g) Opening of M_b.

Digital Histopathology

RoI Detection based on MeshMorphological Operators

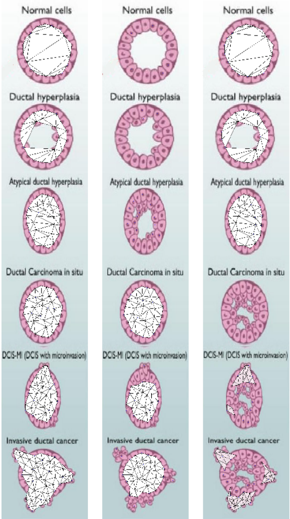
The FlexMim project (FUI-2012-2015) with AP-HP (27 departments)





Digital Histopathology

Tissue Screening with Bio-codes



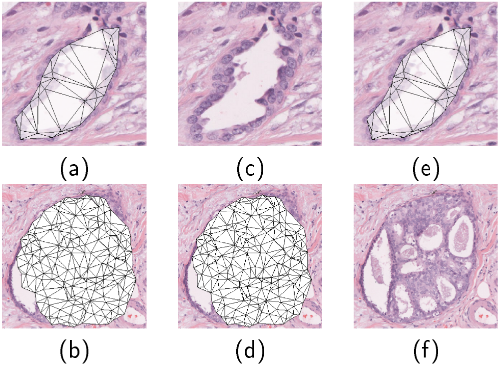
Two structural bio-codes for various breast cancer based on the nuclei organization analysis. The bio-code BC_2 is based on the Euler Number computed over the three mesh representations

Cancer Type	BC1 based on EN,CC,MS	BC2 based on EN
Normal cells	111	101
Ductal hyperplasia	010	000
Atypical ductal	110	101
DCIS	000	110
DCIS-MI	220	112
Invasive ductal	550	115

Digital Histopathology

Tissue Screening with Bio-codes

(a) From left to right : a representation $Del_{\alpha_{opt}}$ of a tubular bio-structure, the opening of order 2 $o^2(Del_{\alpha_{opt}})$ and the difference between the two meshes $Del_{\alpha_{opt}} - o^2(Del_{\alpha_{opt}})$; (b)Idem for DCIS -like bio-structure. These two bio-structures are respectively encoded with the bio-codes *111* and *000*.



Publication (aimed at IEEE Transactions on Medical Imaging) : Nicolas Lomenie, Ludovic Roux, Gilles Le Naour, Shijian Lu, Frédérique Capron and Daniel Racoceanu, Point Set Processing for Histopathological Image Analysis : a visual slide study

Thank You for your Attention

MICO Project
COgnitive MIcroscopy
Daniel Racoceanu
2010-2013

SPIRIT Project
SPatial Interactions In Textures
Thomas Hurtut
2012-2015

Nicolas Loménie, ing, Ph.D.

Laboratoire d'Informatique de Paris Descartes (LIPADE)
Groupe Systèmes Intelligents de Perception (SIP)
http ://w3.mi.parisdescartes.fr/sip-lab/